

LIFE-CYCLE COST OPTIMIZATION FOR FOUNDATION
ENGINEERING

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LIFE-CYCLE COST OPTIMIZATION FOR FOUNDATION ENGINEERING

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The use of reliability-based design (RBD) in foundation engineering offers several advantages over traditional methods. Uncertainties in the load and capacity terms of the design equations can be evaluated rationally using probability theory, and the resultant probability of failure is a comprehensible measure of risk for non-technical people. However, there are a few drawbacks that have not been addressed effectively so far. For example, it is necessary to select target safety levels, typically from the reliability implied in traditional design methods. Also, the resulting reliability is a nominal value that can be significantly different from the true reliability derived from observed failure rates. This issue is rarely mentioned in the literature, but it affects the utility of probability as a communication tool. In addition, costs are not considered explicitly in the design process, and calculations can be excessively complex and time-consuming for simple projects.

A new framework to determine optimum foundation designs that result in minimum life-cycle costs is presented herein. The traditional approach for design optimization is to minimize an objective function, such as the sum of initial costs and expected cost of failure. Existing optimization methods require a number of initial assumptions and use nominal probabilities of failure, leading to inaccurate results. In the proposed framework, the true probability of failure is estimated using Monte Carlo simulation (MCS) or the first order reliability method (FORM). This process considers the variability of input parameters and the probability of “human errors”.

Although optimum design parameters can be obtained with the proposed framework, it would not be used in practice often, because it requires knowledge of reliability methods. A simplified approach is necessary to avoid complex calculations and facilitate its widespread use in ordinary projects. Therefore, a simplified method for approximate economic optimization is proposed.

In an effort to close the gap between research and practice in foundation engineering, all the calculations shown herein can be reproduced in a simple spreadsheet with nonlinear optimization capabilities.

BIOGRAPHICAL SKETCH

The author was born and received basic education in the city of Xalapa, Mexico. In 1995, he moved to Monterrey to study at ITESM (Tecnológico de Monterrey), earning his Bachelor's degree with honors in 1999. After graduation he worked in the construction industry as supervisor in Monterrey and as design engineer in Xalapa.

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In 2006, the National Council for Science and Technology (CONACYT) offered him a fellowship for doctoral studies at Cornell. After earning his Ph.D. degree, he plans to return to ITESM as a full-time professor.

A Maye

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LIST OF ABBREVIATIONS

AASHTO	- American Association of State Highway and Transportation Officials
AFORM	- Advanced first order reliability method
ASCE	- American Society of Civil Engineers
ASD	- Allowable stress design
EPA	- Environmental Protection Agency
FORM	- First order reliability method (Hasofer-Lind method)
FOSM	- First order second moment
GRG2	- Generalized reduced gradient algorithm
HERV	- Human errors as random variables
IBC	- International Building Code
ICC	- International Code Council
LCCO	- Life-cycle cost optimization
LRFD	- Load and resistance factor design
LSD	- Limit state design
MCMC	- Markov chain Monte Carlo
MCS	- Monte Carlo simulation
MFORM	- Mean first order reliability method
MPP	- Most probable (failure) point
MRFD	- Multiple load and resistance factor design
NBCC	- National Building Code of Canada
PE	- Point estimate method
PMA	- Performance measure approach
PRA	- Probabilistic risk assessment
PSF	- Performance shaping factor

QMAS	- Quality management assessment system
RIA	- Reliability index approach
RBD	- Reliability-based design
RBDO	- Reliability-based design optimization
RSM	- Response surface method
SLS	- Serviceability limit state
SORM	- Second order reliability method
ULS	- Ultimate limit state
URBEO	- Unconstrained reliability-based economic optimization
VBA	- Visual Basic for applications
VoSL	- Value of a statistical life

LIST OF SYMBOLS

English Symbols Upper Case

A_{fw}	- Area of formwork
B	- Width of shallow foundation or diameter of drilled shaft
B_0	- Overexcavation distance
C	- Covariance matrix
C_I	- Initial (construction) cost
C_F	- Cost of failure
C_0	- Initial cost when $FS = 1$
COV	- Coefficient of variation
COV_e	- COV of measurement error
COV_F	- COV of load
COV_Q	- COV of capacity
COV_{su}	- COV of undrained strength
COV_ϕ	- COV of friction angle
D	- Depth of shallow foundation or drilled shaft; dead load; design space
E	- Event that an error occurs; earthquake load; error set
$E(\cdot)$	- Expected value
F	- Generic load term
F_N	- Nominal load
FS	- Factor of safety
G^*	- Boundary between safe and failure states
H	- Thickness of footing
I	- Importance factor
L	- Live load; lower triangular matrix in Cholesky decomposition

L_r	- Live roof load
M	- Safety margin
N_c	- Bearing capacity factor for cohesion
N_γ	- Bearing capacity factor for weight
N_q	- Bearing capacity factor for overburden stress
P	- Boundary between error set and error-free set
$P(\cdot)$	- Probability function
Q	- Generic capacity term; wind load
Q_d	- Design capacity
Q_N	- Nominal capacity
Q_s	- Drilled shaft side resistance
Q_t	- Drilled shaft tip resistance
R	- Correlation matrix; rain load; error-free set
S	- Snow load; pedestal width; safe set
T	- Temperature load
$U(\cdot)$	- Uniform probability distribution
V	- Benefit
W	- Self weight of the pile; wind load
Z	- Net benefit objective function

English Symbols Lower Case

a	- Abbreviation for $(1 + COV_Q^2)$
b	- Abbreviation for $(1 + COV_F^2)$
c	- Cohesion
d	- Design parameter
d^*	- Most probable failure point

d_L	- Lower bound of design parameter
d_U	- Upper bound of design parameter
e	- Random variable representing error
g	- Normalized failure cost
$g(\cdot)$	- Performance function
$h(\cdot)$	- Counting function
k	- Normalized cost ratio
m_e	- Mean of error
m_g	- Mean of performance function
m_F	- Mean of load
m_Q	- Mean of capacity
n	- Number of components in a system; standard uniform random variable
p	- Design variable
p^*	- Most probable failure point
p_f	- Probability of failure
q_{ult}	- Ultimate bearing capacity
r	- Reliability
r_{sys}	- System reliability
s	- Standard deviation
s_e	- Standard deviation of error term
s_F	- Standard deviation of load
s_Q	- Standard deviation of capacity
s_u	- Undrained shear strength
u	- Standard normal random variable
v	- Error ratio (nominal over true reliability index)
x	- Random variable

Greek Symbols Upper Case

$\Phi (\cdot)$	- Standard normal probability function
$\Phi^{-1} (\cdot)$	- Inverse standard normal probability function
Ω	- Eigenvectors of the correlation matrix

Greek Symbols Lower Case

α	- Separation coefficient
α_n	- Adhesion factor
β	- Reliability index
β_e	- Reliability index of a single component
β_n	- Nominal reliability index
β_{opt}	- Optimum reliability index
β_{sys}	- Reliability index of a system with multiple components
β_T	- Target reliability index
β_t	- True reliability index
γ	- Total soil unit weight
γ_w	- Water unit weight
ε	- Error term
ζ_{qd}	- Depth correction factor for surcharge term
ζ_{qs}	- Shape correction factor for surcharge term
$\zeta_{\gamma d}$	- Depth correction factor for friction term
$\zeta_{\gamma s}$	- Shape correction factor for friction term
η	- Load factor
κ_D	- Ductility factor
κ_I	- Importance factor
κ_R	- Redundancy factor

λ	- Vector of eigenvalues of the correlation matrix
μ	- Mean vector
ν	- Poisson's ratio
ρ	- Pearson's correlation coefficient
σ	- Standard deviation of the population
$\varphi(\cdot)$	- Standard normal probability density function
ϕ	- Angle of friction
ϕ'	- Effective stress angle of friction
ψ	- Resistance factor

CHAPTER 1

BACKGROUND

1.1 Introduction

Selection of target safety levels in geotechnical engineering has long been an important research topic. Unfortunately, it has not received sufficient attention by the entire professional community. However, there is good reason to spend more time and resources on this task. The primary objective of geotechnical designers is to specify the location, dimensions, materials, and construction methods required to build functional and safe structures in a cost-effective manner. In other words, designers are expected to consider at least three fundamental factors: safety, serviceability, and economics. These three factors are related closely, but we often separate them to simplify the design process. Constructibility is an additional factor to consider.

Safety of the users, contents, and the structure itself is almost always the most important requirement, at least for building and bridge structures. In general, systems can not function as intended when their safety is not guaranteed. Even if failure does not occur, the perception of risk can harm the usability of a structure. Regarding economics, any type of failure or function loss will result in additional costs to the owner(s), user(s), and/or society. On the other hand, overly conservative designs can reduce the risk of failure but will result in an unnecessary additional cost to the owner. As a result, engineers must balance designs to provide reasonable safety levels at a reasonable cost. Unfortunately, there is no rational design method to achieve this fundamental balance; engineers must rely on precedence, their experience, and good judgment.

The traditional design process is a “trial and error” approach in which designers propose a feasible solution and then check if it satisfies safety and serviceability requirements. If the initial estimate satisfies both requirements and is not excessively conservative (too expensive), the solution is adopted. If not, the solution is modified, and the process is repeated until all design requirements are met.

Another fundamental and often overlooked issue is that safety and serviceability requirements also are empirical. All of the existing structural design methods (allowable stress design, factor of safety, load and resistance factor design, reliability-based design, etc.) require target safety levels that have been calibrated empirically, largely based on observed failure rates. Empirical methods are not necessarily less reliable or less accurate than rational approaches, especially since they are supported by actual observations of structural systems over a long period of time. However, rational methods should be able to reproduce and explain empirical results. There are a number of disadvantages that must be considered when evaluating the validity of different design methods. A broader discussion on empirical and rational methods is presented in Chapter 2.

1.1.1 Motivation

The topic of this dissertation was motivated by the desire to improve current foundation design methods, which have inherent limitations. None of the available methods considers economic variables explicitly, even though optimum target safety levels and failure cost are closely related. Another problem is caused by variations of economic conditions over time. Empirical methods need a relatively long time to adjust safety levels based on observations. Unfortunately, globalization of markets,

technological developments, climate change, and other phenomena can change economic conditions rapidly. A rational framework is required to make optimal use of available resources taking into account the uncertain nature of the problem.

Probabilistic design methods have evolved since the mid-1900s to address the uncertainties related to engineering design. In geotechnical engineering, these methods slowly are entering the curricula of some academic programs and are being used in the calibration of building codes, but their use in routine practice is still limited. A study by the National Research Council (2006) found that there are significant knowledge gaps in geotechnical engineering practice. Among other findings, they recognize the need to improve: (a) geotechnical characterization technology, (b) quantification of uncertainties, and (c) methods to assess the impact of those uncertainties.

Additionally, some authors recognize the need to include costs directly in the design process. Many recent events have shown that the supply of some products can change drastically in short periods of time. Overpopulation pressures, growth of large economies such as China and India, and international conflicts, among other factors, have increased the demand for energy and construction materials. Consequently, the cost of reinforcing steel in the U.S. had a 40% increase between 2006 and 2009. Oil and gas prices also have had large changes in short periods of time. As the world approaches peak oil production and new sources of renewable energy become available, sudden changes in energy prices are expected to continue. It seems obvious that costs must be included, not only to select safety levels, but also to optimize designs.

Another related problem is the emission of greenhouse gases that cause global warming. The main source of greenhouse gases is the combustion of fossil fuels for

transportation and energy production, but other processes, such as the fabrication of construction materials, also contribute. According to the Environmental Protection Agency (2009), the steel and cement industries are the largest producers of carbon dioxide (CO_2) for non-energy uses. In 2007, these two industries generated 121.9 million metric tons of CO_2 in the U.S., which is equal to 91% of emissions for non-energy uses or 2% of total CO_2 emissions. Although the construction industry represents a small percentage of total greenhouse gas production, environmental impact is a hidden cost typically not considered in cost-benefit analyses of civil infrastructure projects.

As a consequence, a rational framework for global cost optimization should decrease the expected life-cycle cost of any construction project. Economic savings in a single project can be small or unnoticeable because the cost of failure will not occur in most cases. However, for a large number of projects, the cumulative economic savings should be significant. If minimization of expected global cost is not a motivation for small building owners, it should be an incentive for large owners, building code officials, insurance companies, governments, investors, etc.

1.1.2 Development of Foundation Design Methods

Understanding the behavior of geomaterials (soils and rocks) has helped greatly to build safe and relatively economical structures. Before the introduction of modern soil mechanics, engineers relied upon empirical design methods that yielded conservative designs in some cases and unsafe structures in others. The old and new theories used in geotechnical practice (e.g. earth pressure theories, effective stress concept, consolidation theories, bearing capacity equations, etc.) enabled engineers to make

better predictions of the capacity of structures, which effectively reduced uncertainty in their performance and increased their reliability.

However, the behavior of real structures is not deterministic, even when the most sophisticated theories are used. Our models can not be perfect since they are simplified approximations of material behavior under ideal conditions. For instance, we commonly use homogeneous and isotropic models when natural geomaterials rarely are either. Moreover, loads acting on structures depend on environmental conditions. The interactions that govern atmospheric and geologic processes are still too complex to predict weather conditions or seismic activity consistently or accurately. Therefore, environmental loads have to be estimated typically from existing records with the corresponding degree of uncertainty. Finally, both the resistances and loads on structures are affected by human activities. If natural processes are difficult to model, human behavior is even more complex and unpredictable.

Considering these conditions, the traditional design method to ensure adequate performance of foundations is the global safety factor approach. The safety factor for a given failure mode is defined as the ratio of the available capacity to that required. One of the reasons for the widespread use of this method in many branches of engineering is its simplicity. To calculate a factor of safety, designers have to derive loads and capacities from a set of design variables, but no probability calculations are required. The target safety factors used in practice have been calibrated based on many years of experience, requiring higher factors when the uncertainty is large and when the consequences of failure are grave. Although there are general guidelines and recommendations in the literature for the selection of safety factors, designers can have different opinions depending on their knowledge, experience, confidence, or risk

aversion. Moreover, the safety factor for a particular failure mode is not an invariant quantity. The resultant safety factor depends on the definition of the acting and resisting terms (See Section 2.2).

Although the global safety factor approach has served well for a long time, today it is possible to use probabilistic methods to evaluate explicitly the known uncertainties in the design. Essentially, it is possible to propagate the uncertainties of the design parameters to the predicted performance of a structure. Probabilistic methods, such as reliability-based design (RBD), are being implemented in both structural and geotechnical design practice because they offer several advantages over traditional deterministic methods. Despite the progress in probabilistic methods, it is still necessary to select a target safety level. This task is not trivial because the definition of acceptable risk in civil infrastructure projects has always been controversial. Even if a maximum acceptable risk could be defined clearly and accepted by the professional community, there is no rational argument to use such a value as a design target.

Intuitively, the reliability of a structure should be a function of the associated costs and benefits. It does not make sense to use preset target reliabilities regardless of the size or function of the structure. This intuitive idea is supported by decision theory, which provides a criterion to select the best alternative in the presence of uncertainty. Unfortunately, this approach has not reached routine practice yet, because some difficulties arise in the process. Selection of target safety levels requires more attention from both the academic and professional communities in civil engineering.

1.1.3 Probability Concepts

Before addressing RBD and cost optimization, it is appropriate to clarify some concepts used in reliability methods. Surprisingly, a large fraction of RBD criticism comes from wrong interpretations of probability theory, not from the assumptions or the results.

Probabilistic methods emerged as a way to predict outcomes of unknown processes or known processes with uncertain input parameters. The origin of probability theory is generally dated back to the 17th century, when Pascal and de Fermat solved a gambling problem proposed by de Méré. However, the term probability was already used at the time to describe the quality that something could be proven by an expert. The word came from the Latin *probabilis*, and it was interpreted as an opinion or belief (Vick 2002).

Today, probability theory is a branch of mathematics with a formal set of laws or axioms defined by Kolmogorov in 1933 (Hendricks et al. 2001), but we still use probability as a measure of belief. In fact, there are at least two interpretations of probability: the frequentist and the Bayesian. Vick defined these two concepts as:

Relative frequency approach: The probability of an uncertain event is its relative frequency of occurrence in repeated trials or experimental sampling of the outcome.

Bayesian, degree-of-belief approach: The probability of an uncertain event is the quantified measure of one's belief or confidence in the outcome, according to their state of knowledge at the time it is assessed.

In civil engineering, it is necessary to use both interpretations of probability because most civil infrastructure projects are unique. It is impossible to determine the relative frequency of failures when only one structure exists.

The concept is similar to the probability of a particular event in weather forecasting. The probability of rain for a given day and location corresponds to the degree-of-belief approach. However, a good meteorologist should be correct half of the time that he or she predicts a 50% chance of rain. In the same manner, if a thousand distinct structures are designed with a probability of failure of one percent, then approximately ten structures should fail during their design lives.

Although the probability of failure of a unique structure belongs to the degree-of-belief category, it is the result of a logical calculation process that follows a set of formal mathematical rules. Also, the probability distributions of the input random parameters are not subjective; they are inferred from the statistical properties of representative samples. Sometimes the term “logical approach” is used to distinguish this interpretation from an entirely subjective, personal belief. A personal assessment of the likelihood of an event is commonly called “subjective probability”.

1.1.4 Risk Analysis and Decision Theory Concepts

As mentioned previously, the reliability of a structure should be proportional to the potential losses. This intuitive notion is defined formally as engineering risk, which is equal to the probability of failure multiplied by the losses or cost of failure.

$$\text{Risk} = (\text{Probability of failure}) \times (\text{Losses because of failure}) \quad (1-1)$$

There are other definitions of risk depending on the context, but Equation 1-1 is generally accepted in engineering and other disciplines in which a quantitative assessment is necessary. Therefore, if the losses are evaluated in monetary terms, risk is equivalent to the expected cost of failure.

Risk analysis is a discipline that focuses on minimizing damages from failures in complex engineering processes. It goes beyond reliability analysis, because it includes risk identification, analysis, and mitigation strategies. However, its main focus is in engineering processes rather than engineering structures. The probabilities of failure generally are evaluated using probability theory, in which a probability of failure and the associated losses are estimated for each potential failure mode to obtain a nominal risk. Other empirical methods also are used to estimate risks from complex processes such as human operations. In this way, the global risk of several alternatives can be compared. Finally, risks can be monitored and mitigated during operation using different risk management techniques.

In contrast to risk analysis, the goal of decision theory is not to minimize risk but to maximize benefits. It is not enough to select the option with the lowest risk, because other costs are not considered. Construction costs seldom are considered as a risk, but they certainly affect the expected benefit of the facility. For new structures, initial costs usually are proportional to the reliability (or inversely proportional to the expected cost of failure).

From an economic standpoint, if all the benefits and consequences of failure are quantified in monetary terms, it is possible to find the optimum reliability level associated with the maximum expected net benefit. Even when it is not possible to assign monetary values to all the factors, decision theory makes use of utility functions

to evaluate different options rationally. The framework presented in Chapter 5 applies decision theory and cost-benefit analysis to optimize geotechnical designs based on their life-cycle cost.

1.2 Objectives

The idea of using optimization techniques and cost functions in civil engineering is not new. Since the 1960s at least, some authors have proposed the use of decision theory in structural design (Turkstra 1967). Figure 1-1 is an illustration of the simplified relationship between safety levels and life-cycle costs. When safety levels are excessively low, initial costs are low, but the probability of failure and the expected cost of failure are high.

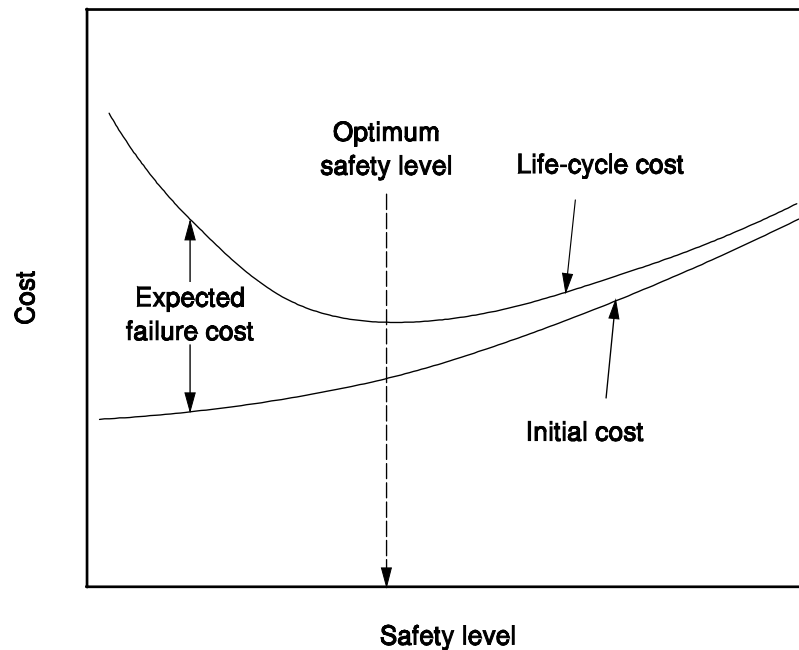


Figure 1-1 Simplified relationship between safety level and life-cycle cost (adapted from Phoon et al. 2000)

However, when safety levels are too high, the probability of failure decreases, but the total cost increases because of excessive initial costs. There is an optimum safety level that produces minimum life-cycle costs.

Unfortunately, these concepts rarely are used in actual design projects for a number of reasons. Even simplified RBD methods, such as load and resistance factor design (LRFD), have found some opposition by practitioners. Probably the most common argument against implementation of RBD is that the traditional approach has been used for many years and failure rates are acceptable. It is true that current failure rates are relatively low and accepted by society. However, this argument implies that design methods can not be improved, which is false. Optimization techniques can reduce initial costs while maintaining acceptable failure rates.

The purpose of this study is to develop a rational framework that applies decision theory for foundation design, addressing some of the limitations of available methods. Similar to other numerical methods, the results must be evaluated carefully to determine if they are applicable to the unique conditions of the project. As always, engineers must use their judgment and experience to estimate appropriate design properties, models, construction procedures, and costs.

To illustrate the proposed design process, consider Figure 1-2. In theory, optimum designs should be determined from a set of design parameters, their statistical properties, and expected costs during the life of the structure. Calculations include limit state equations to predict failures, an objective cost function to be minimized, and an optimization algorithm. The results of this process are the final design and its reliability for minimal life-cycle costs. In fact, the reliability is an intermediate result

not needed after the final design is available, but it can be useful to compare similar projects and validate the results.

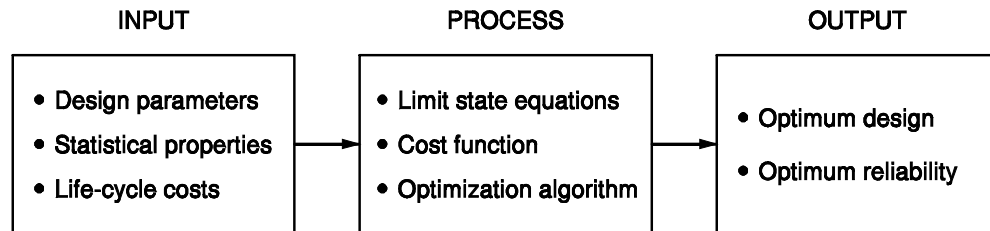


Figure 1-2. Diagram of the proposed life-cycle cost optimization method

In some cases, the additional effort required by full optimization analyses is not justified, especially for routine projects. Simplified methods are necessary for practical use in routine design. Therefore, another objective of this work is to present a simplified method to apply decision theory in foundation design.

In summary, the objectives of this research are to:

- demonstrate the importance of life-cycle costs in the selection of target safety levels
- develop a rational framework for life-cycle cost optimization using a simple spreadsheet with nonlinear optimization capabilities
- estimate the true probability of failure, including the possibility of human errors
- present a simplified procedure that includes life-cycle costs for foundation design

1.3 Organization of Document

Each chapter herein presents a different aspect of the design optimization problem based on cost. Chapter 2 provides an overview of current design methods and building code specifications, focusing on cost considerations and target safety levels. Chapter 3 describes common optimization techniques found in the literature and their limitations. It is safe to say that such techniques are not used in practice; their use is mostly restricted to research or very special projects. Chapter 4 discusses the effects of “human errors” in structural reliability. Two methods found in the literature and two new quantitative approaches are presented to calculate the true probability of failure. Chapter 5 describes the proposed optimization framework for a single failure mode, including three different methods to calculate the probability of failure. A shallow foundation design and a drilled shaft design serve as examples of the framework, and the results are compared with simplified solutions for nominal and true reliabilities. Finally, a sensitivity analysis reveals the most significant parameters for each problem. Chapter 6 shows an alternative simplified approach to include life-cycle cost in the process. The same example problems presented in Chapter 5 are employed using the simplified approach. Chapter 7 contains a summary of the results, conclusions, and proposed future research to complement this study.

CHAPTER 2

REVIEW OF RELEVANT FOUNDATION DESIGN METHODS

2.1 Introduction

In this chapter, traditional geotechnical design methods are reviewed, and their strengths and limitations are noted. These methods have been used successfully for many years in foundation engineering and have been calibrated for a large number of conditions. Although probabilistic methods will be used more frequently in the future, traditional methods will be crucial for calibration or comparison and for a broad range of more routine projects where complex design methods are unnecessary.

The empirical approach used to calibrate safety levels is very convenient, because it is based on actual behavior of structures. In principle, it can take into account any type of failure, including natural load and resistance variability, model uncertainty, and errors in design and construction. Consequently, the average rate of foundation failures is relatively low and is accepted by society. In some cases, the general methods are adjusted to incorporate local knowledge about the prevailing conditions in a region. Unfortunately, empirical methods can not discern the source of the problems, having to assign a uniform safety level for average cost and variability.

Rational methods try to evaluate consistently the probability of failure by different causes and allow adjustment of the target safety levels for particular cases using the information available. However, the risk of inaccurate results can be higher in rational methods, because the results are not paired with empirical evidence. In any type of engineering analysis, there must be a compromise between rational arguments and

empirical evidence. Foundation design is no exception to this rule. The better that observed phenomena can be explained using rational arguments, the better our design methods will be.

Each design method will be summarized focusing on how uncertainty and life-cycle costs are addressed. In the last part of this chapter, an overview is presented of the methods used in different building codes.

2.2 Traditional Global Safety Factor

The safety factor of a structure or component for a particular failure mode is defined as the ratio of the available capacity over that required (Eq. 2-1).

$$FS = \frac{Q}{F} \quad (2-1)$$

In general, the required capacity, Q , is equal to the maximum demand or load, F , acting on the structure during its design life. Considering a perfect model, failure should occur when the safety factor is less than one ($FS < 1$). This is one of the simplest ways to express the relationship between the load and the resistance for a particular failure mode, although sometimes defining those two terms is not straightforward. The safety factor has been preferred over the safety margin, M , defined in Equation 2-2, probably because it is a dimensionless term.

$$M = Q - F \quad (2-2)$$

Few authors recommend safety factors in their publications. Some only present sample calculations using typical values, but they do not make specific recommendations,

because there is no rational basis for their selection. The only advice frequently found in the literature is very sensible: be cautious and use good judgment during the selection of safety factors. Nonetheless, some typical ranges generally are accepted for each failure mode and soil type.

During the development of modern soil mechanics, engineers and researchers applied new theories using large safety factors to be on the conservative side. For some engineers, it was obvious that a larger safety factor was required when the uncertainty in the input parameters (soil properties and loads) was large.

In the first edition of “Soil Mechanics in Engineering Practice”, Terzaghi and Peck (1948) made the following remark regarding the bearing capacity of driven piles:

The agreement between the real ultimate bearing capacity and that computed on the basis of the Engineering News formula is hardly satisfactory... Of greater significance, however, was the fact that individual values of the real bearing capacity ranged from 0.3 to 2.8 times the computed values.

As a result, the safety factor recommended in this text for use with the Engineering News formula was 6, so that the true safety factors lie between 2 and 17.

Another important factor in the selection of target safety values is the compatibility of the foundation with the superstructure. It seems unreasonable to design a foundation for a reliability lower than that of the superstructure, because a resistance failure will result in the loss of both components. Terzaghi and Peck (1948) stated:

First, the factors of safety of the foundation with respect to the breaking into the ground should not be less than 3, which is the minimum factor of safety customarily specified for the design of the superstructure.

This argument is not entirely correct, because the reliability of two components with the same safety factor is not necessarily equal. The variability in the resistance of foundations typically is higher than the variability in the resistance of structural members. Furthermore, the cost to achieve the same reliability in the substructure and the superstructure may be prohibitive (See Chapter 6).

In any case, the argument is no longer valid because the safety factor of structural components typically is less than 2 in modern building codes, while the safety factor of foundations remains between 2 and 3.

Tables 2-1, 2-2, and 2-3 summarize some safety factors found in the literature for geotechnical design.

Table 2-1 shows that the recommended safety factors for the same failure mode are higher when the variability of the load is higher (extreme events). Similarly, the recommended values in Table 2-2 depend on the consequences of failure and variability of the resistance (extent of soil exploration).

It is assumed that the variability of the resistance parameters is lower when the site exploration is complete. In some sense, target safety factors are adjusted intuitively to achieve similar reliability levels. In fact, remarks a, b, and c in Table 2-2 are empirical means to achieve lower expected life-cycle cost of structures.

In Table 2-3, the recommended safety factor is not equal for all failure modes. This empirical result is not caused by different consequences of failure or higher variability of the input parameters. A probabilistic analysis can show that the variability of the safety factor also depends on the form of the equations used.

Table 2-1. Some recommended safety factors in the literature

Problem	Failure mode and conditions	Safety factor	Source
Shallow foundations	Bearing capacity, clays under normal loads	3	a
	Bearing capacity, clays under extreme loads	2	a
	Bearing capacity, dead plus live load	3	c
	Bearing capacity, temporary live load (earthquake, wind, etc.)	2	c
Deep foundations	Total capacity, permanent loads	3	c
	Total capacity, temporary loads	2	c
	Tip resistance of driven piles, SPT correlation	3	c
	Side resistance of driven piles, SPT or CPT correlation	3	c
	Side resistance, uplift, pile group, granular and cohesive soils, sustained loads	3	c
	Side resistance, uplift, pile group, cohesive soil, short term load	2	c
Retaining walls	Overall stability	2	c
	Overturning, static	1.5	c
	Sliding, static	1.5	c & d
	Sliding, overturning combined static and earthquake loads	1.1-1.2	c
Slope stability	Permanent cut slopes	1.5	c
	Temporary cut slopes	1.3	c
	Permanent earth berms	2	c
	Temporary earth berms	1.5	c
	Embankments for worst conditions	1.5	d
Excavations	Bottom heave in permanent excavations	2	c
	Bottom heave in temporary excavations	1.5	c
	Bottom heave in soft clays	1.5	d
	Piping or heave in sands	1.5-2	b
	Buckling of struts	2	d

Sources:

- a. Peck et al. 1974, Foundation Engineering.
- a. NAVFAC 1986a, Design Manual 7.01 - Soil Mechanics.
- b. NAVFAC 1986b, Design Manual 7.02 - Foundations and Earth Structures.
- c. d. Terzaghi and Peck 1948, Soil Mechanics in Engineering Practice.

Table 2-2. Minimum safety factors for design of shallow foundations (Vesic 1975)

Category	Typical structures	Characteristics of category	Soil exploration	
			Complete	Limited
A	Railway bridges Warehouses Blast furnaces Hydraulic Retaining walls Silos	Maximum design load likely to occur often; consequences of failure disastrous	3	4
B	Highway bridges Light industrial and public buildings	Maximum design load may occur occasionally; consequences of failure serious	2.5	3.5
C	Apartment and office buildings	Maximum design load unlikely to occur	2	3

Remarks:

- For temporary structures, these factors can be reduced to 75 percent of the above values. However, in no case should safety factors lower than 2.0 be used.
- For exceptionally tall buildings, such as chimneys and towers, or generally whenever progressive bearing capacity failure may be feared, these factors should be increased by 20 to 50 percent.
- The possibility of flooding of foundation soil and/or removal of existing overburden by scour or excavation should be given adequate consideration.
- It is advisable to check both the short-term (end-of-construction) and long-term stability, unless one of the two conditions is clearly less favorable.
- It is understood that all foundations will be analyzed also with respect to the maximum tolerable load and differential settlement. If settlement governs the design, higher safety factors should be used.

Table 2-3. Values of minimum total safety factors (Meyerhof 1984)

Failure type	Item	Safety factor
Shearing	Earthworks	1.3 – 1.5
	Earth retaining structures, excavations	1.5 – 2
	Foundations	2 – 3
Seepage	Uplift, heave	1.5 – 2
	Exit gradient, piping	2 – 3

Also note that Tables 2-2 and 2-3 specify *minimum* safety factors, because the cost of a deficit in capacity (failure) is much larger than a surplus (overdesign), as will be shown in Chapter 5.

Not surprisingly, the global safety factor method has been criticized by many authors. Lumb (1970) called the selection of safety factors “... *a personal choice, subjective and quite often arbitrary.*” Perhaps the most serious drawback is that the safety factor for a particular failure mode is not unique because it depends on the definition of the demand and capacity terms. Kulhawy (1996) presented an example in which five different uplift capacities were calculated for a drilled shaft with a safety factor supposedly equal to three. All the results were correct for the particular assumptions and definitions used. Also, the nominal values of the capacity and load terms are not standard. Engineers could select practically any design value: the mean, some quantile, a conservative average, or any other number.

Regarding cost, the safety factor method does not provide any direct guidance to optimize designs. The foundation designer must select an adequate type of foundation, geometry, and construction method that satisfies safety, service, and cost requirements. Obviously, experience and judgment are very important to achieve

economic designs. Some of the early cost optimization studies in structural engineering used optimization algorithms with the global safety factor as a constraint. However, this process is not part of the traditional safety factor approach. A more detailed description of cost optimization techniques is presented in Chapter 3.

2.3 Reliability-Based Design

Reliability-based design (RBD) is a more recent design method in which the variability of the performance and the probability of failure can be evaluated rationally. The most common approach to evaluate uncertainties is the use of probability theory, although other models have been studied, such as possibility theory (Nikolaidis et al. 2004). In RBD, engineers do not have to assume that the design parameters are deterministic; they can calculate a nominal probability of failure and compare it to a predetermined target value. If the nominal probability of failure is greater than the target, then the design is modified and the process is repeated until the result is acceptable.

The overview presented in this subsection is intended to summarize the basic assumptions and discuss the advantages and limitations of RBD. For a more detailed description of RBD methods, other sources are available (e.g. Thoft-Christensen and Baker 1982, Melchers 1987, Harr 1987, Baecher and Christian 2003, Raizer 2004).

To predict the behavior of geotechnical structures, engineers typically use the limit state concept. In limit state design (LSD), any structure under specific assumed conditions belongs to one of two possible states: failure or no failure. And failure modes are classified as either resistance or deformation, also called ultimate limit state

(ULS) and serviceability limit state (SLS), respectively. The philosophy of LSD, as described by Phoon et al. (2000), can be summarized in three steps:

- (a) identify potential failure modes or limit states
- (b) apply separate checks on each limit state
- (c) show that the occurrence on each limit state is sufficiently improbable

Limit state equations or performance functions generally have the form of the safety margin, given below, but the safety factor can be used as well:

$$g(x) = Q(x) - F(x) \quad (2-3)$$

in which g = performance function, Q = generic capacity or resistance, F = generic load, and x = vector of variables that can be deterministic or random.

In a perfect model, failure should occur when $g(x)$ is less than or equal to zero. When the safety factor, FS , is used instead of the safety margin, the performance function is equal to $(FS - 1)$.

The term reliability has several definitions but, in engineering, reliability is the probability that a unit performs adequately for a specific period of time. Consequently, the probability of failure, p_f , is the complement of the reliability ($p_f = 1 - r$).

Since the probability of failure for modern structures is a very small number, the reliability index, β , (sometimes called the safety index) is used as an alternative way to quantify safety, as given below:

$$\beta = \frac{m_g}{s_g} \quad (2-4)$$

in which m_g and s_g = mean and standard deviation of the performance function, respectively, as shown in Figure 2-1.

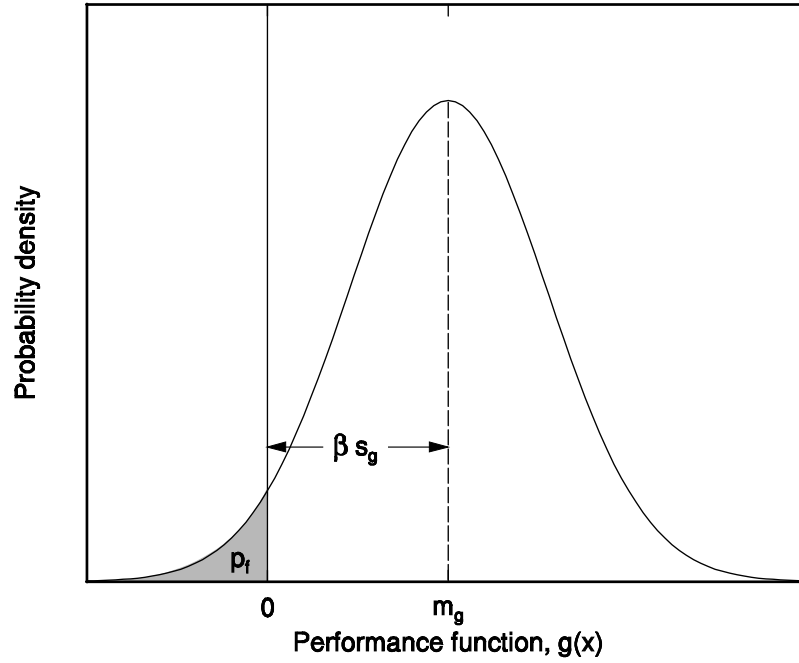


Figure 2-1. Probability distribution of performance function

The reliability index can be interpreted as the distance, measured in standard deviations, from the mean of the performance function, m_g , to the limit state ($g = 0$). Therefore, if the performance function is normally distributed, the probability of failure can be computed as follows:

$$p_f = \Phi(-\beta) \quad (2-5)$$

in which $\Phi(\cdot)$ = cumulative standard normal distribution function.

The values obtained from Equation 2-4 are valid by definition, regardless of the distribution of the performance function. However, if the reliability index is calculated

from the probability of failure (solving for β in Eq. 2-5), the result is an approximation because it implies that the performance function is normally distributed.

When the performance function has the form of the safety margin, and the load and resistance terms are independent, normally distributed, random variables, the reliability index is computed as follows:

$$\beta = \frac{m_Q - m_F}{\sqrt{s_Q^2 + s_F^2}} \quad (2-6)$$

in which m_Q and m_F = means of the capacity and demand, and s_Q and s_F = standard deviations of the capacity and demand, respectively.

A similar equation can be used when the Q and F are independent, lognormal random variables, as given below:

$$\beta = \frac{\ln \left[m_Q / m_F \sqrt{(1 + \text{COV}_F^2) / (1 + \text{COV}_Q^2)} \right]}{\sqrt{\ln \left[(1 + \text{COV}_F^2)(1 + \text{COV}_Q^2) \right]}} \quad (2-7)$$

in which $\ln(\cdot)$ = natural logarithm function, and COV_Q and COV_F = coefficients of variation of the capacity and demand, respectively.

Equation 2-7 was derived by combining Equation 2-6 and the formulas for the first two moments of lognormal variables, as given below:

$$m_{\ln Q} = \ln(m_Q) - \ln \sqrt{1 + \text{COV}_Q^2} \quad (2-8)$$

$$s_{\ln Q} = \ln(1 + \text{COV}_Q^2) \quad (2-9)$$

Another relevant issue is that the relationship between the reliability index and the probability of failure is not linear. Table 2-4 shows the probability of failure for different reliability indices obtained from Equation 2-5.

Table 2-4. Relationship between reliability index and probability of failure

Reliability index, β	Probability of failure, p_f
0	0.5
1	0.16
2	0.023
3	0.0014
4	0.000032

It is convenient to express safety levels in terms of the reliability index for many reasons. A small range of values can express probabilities of failure that vary over several orders of magnitude. Also, common target values are easy to remember, because the range is similar to some safety factors used in geotechnical engineering. In general, reliability indices less than one are unacceptable, but values between two and three are not unusual.

However, it is easy to disregard the variability of the key result, which is the probability of failure. For example, a 10% reduction in the reliability index (from 3 to 2.7) may seem insignificant, but it implies an increase of 150% in the probability of failure (from 0.14% to 0.34%).

Although RBD is applied mostly in ULS foundation design, it also can be used for SLS and economic optimization in foundation engineering (Wang and Kulhawy 2008b, Zhang and Phoon 2006).

2.3.1 Reliability Approximations

In reality, the demand and capacity terms of the performance function rarely have simple, independent probability distributions. Therefore, the exact probability of failure is the integral of the joint probability density function of the design parameters over the failure domain:

$$p_f = \int_{g < 0} f(x) dx \quad (2-10)$$

in which g = performance function, $f(\cdot)$ = joint probability density function, and x = vector of all the random variables.

In most cases, the joint probability density function of the random variables is not available, and the limits of integration of Equation 2-10 are difficult to determine. Under those conditions, the only option is to obtain an approximation of the probability of failure using Equations 2-6 and 2-7 or to use numerical methods such as Monte Carlo simulation (MCS), first order reliability method (FORM), first order second moment (FOSM), point estimate method (PE), etc. All of these methods have been studied and applied to geotechnical engineering problems (e.g. Baecher and Christian 2003), but the application to life-cycle cost optimization (LCCO) has been limited. (See Chapter 3)

The point estimate method is an elegant approach proposed by Rosenblueth in 1975 and was applied to geotechnical problems by some researchers during the 1980s and 1990s (Harr 1987, Li 1992, Christian and Baecher 1999). Unfortunately, results can be highly inaccurate for some types of functions, especially when the coefficients of variation of the input parameters are large. Since the coefficients of variation of

geotechnical parameters tend to be higher than other variables, this method is not applied in current design practice. Therefore, the point estimate method is not included in this study.

Another reliability method used in geotechnical engineering for calibration of simplified probabilistic equations is the mean value FOSM (Duncan 2000). It is based on the first terms of a Taylor series expansion of the performance function. The method is relatively straightforward, and only the first two statistical moments of the design parameters (mean and standard deviation) and the correlation matrix are required. The value of β is computed as follows:

$$\beta = \frac{g(m_x)}{\sqrt{\left[\frac{\partial g}{\partial x_i} \right]^T C \left[\frac{\partial g}{\partial x_i} \right]}} \quad (2-11)$$

in which m_x = vector of mean values of x , $\partial g / \partial x_i$ = vector of gradients of the performance function evaluated at the mean value of x , and C = covariance matrix.

Paikowsky (2004) reported that resistance factors for ULS design of deep foundations calibrated with FOSM were on average 10% lower than the results from FORM. Notice that a 10% difference in the resistance factors does not imply the same difference in the probability of failure. In general, FOSM results are accurate only when the performance function is approximately linear. Christian and Baecher (2001) noted that using the safety margin (instead of the safety factor) produces consistent and accurate results in FOSM calculations. It is true that using the safety margin sometimes can improve the accuracy of FOSM, because the division in the safety factor equation is replaced by a subtraction. However, if the equations used to calculate the load and resistance terms are highly nonlinear, the error can be

unacceptable even when using the safety margin. FOSM is still a good option for simplified models with linear functions, but FORM or MCS do not have this limitation and require only a small additional effort.

FORM is another approximation to compute structural reliability that was developed by Hasofer and Lind in 1974. The idea behind FORM is to find the minimum statistical distance between the mean of the design variables, m_x , and the failure surface in the standard normal space. The point on the failure surface closest to the mean point is called the most probable failure point or MPP (Figure 2-2).

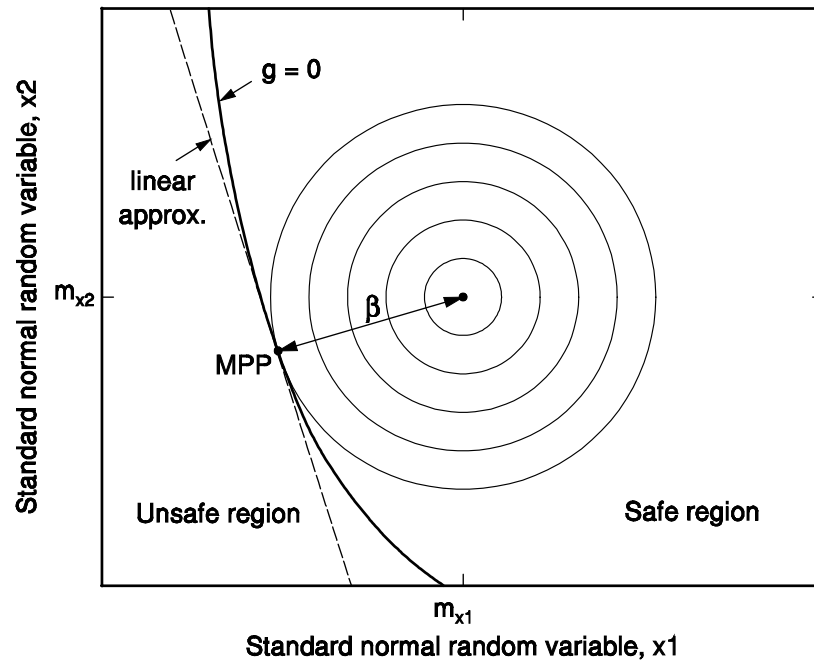


Figure 2-2. Illustration of FORM elements

Rackwitz and Fiessler (1978) developed an iterative method to find the distance between the two points, which is equal to the reliability index, β . Low and Tang (1997) showed that reliability calculations are equivalent to optimization problems and can be solved using a spreadsheet with optimization capabilities such as Excel Solver.

Although the reliability from FORM is an approximation, it is not sensitive to the shape of the performance function. Results using the safety factors or safety margins are practically the same. Also, for relatively high reliability indices ($\beta > 2$), the linear approximation of the performance function is sufficiently accurate in most cases. The reliability index in FORM is calculated as follows:

$$\beta = \min \sqrt{(\mathbf{x} - \mathbf{m}_x)^T \mathbf{C}^{-1} (\mathbf{x} - \mathbf{m}_x)} \quad (2-12)$$

Monte Carlo simulation is a computationally intensive method that does not require major assumptions. This method determines the probability of failure from the relative number of failures that occur when the design parameters are simulated randomly according to their probability distributions. Each random variable can have any probability distribution, and the performance function is not assumed to be linear. One of the requirements to obtain accurate results is a sufficiently large number of simulations. A rule of thumb indicates that $10/p_f$ simulations should be used (Phoon 2008). For example, if the reliability index is 4, then the probability of failure is 0.000032, and the required number of simulations is greater than 300,000. There are some techniques to decrease the number of required simulations, but the complexity of the problem increases. Honjo (2008) discussed the most common sampling techniques, including subset Markov chain Monte Carlo (MCMC).

Reliability-based design, like all traditional design methods, is an iterative process. The designer must make an initial estimate of the solution (a point in the design space) and then calculate the reliability. If the reliability index is close to the target value, then the proposed solution is accepted. If not, the solution is modified, and the process is repeated until the computed reliability is acceptable. Although RBD provides a

more rational framework to achieve uniform safety levels, the selection of target reliabilities is still a “fine art”.

2.3.2 Target Reliability

Selection of target reliabilities used in probabilistic analyses is a problematic subject, but it is a crucial step, because the final design depends greatly on the values chosen. In general, there are four ways to determine target safety levels: (a) reliability implied in current design methods, (b) reliability from observed failure rates, (c) risk accepted by society, or (d) cost-benefit analyses. Every method has advantages and limitations that must be considered.

Reliability implied in current design methods (ASD) is the most common procedure for several reasons. Allen (2005) recommended using this method for calibration of simplified probabilistic methods (e.g. LRFD) for foundation design. Also, target safety levels are consistent with traditional design methods, making easier the transition to probabilistic techniques. Allen argues that reliability implied in ASD must be close to optimum levels, because it has been adjusted empirically over time. However, traditional design methods do not consider site-specific information. Therefore, the resulting safety levels can be close to optimum only for average conditions.

Phoon et al. (1995) described the process in four steps:

- (a) select a set of representative design problems
- (b) determine an acceptable solution to each problem based on existing methodology, such as the allowable stress method

(c) evaluate the probability of failure for each design

(d) based on the results obtained in the previous step, select a target reliability

Most of the currently recommended values were obtained with this method. For example, Meyerhof (1984) noted that the nominal lifetime probability of failure implied in foundation designs for ULS varies from 0.0001 to 0.01 (β from 2.5 to 3.5). Note that the use of reliability methods with fixed target reliability values impedes cost optimization. This constraint is one of the reasons why probabilistic methods have found some opposition in the professional community. The rigidity of probability theory, combined with fixed target reliability values, does not allow adjustments of the design based on the engineer's judgment. The cost minimization approach proposed herein does not require a pre-defined target safety.

The second approach is the reliability from observed failure rates. A number of authors have pointed out that this method is difficult to use in practice because the observed failure rates are one or two orders of magnitude higher than the theoretical or nominal values (Brown et al. 2008). The difference is caused by possible human errors during design, construction, or use of the structure. Some studies list observed failure rates for different types of structures or industries. Baecher (1987) showed typical annual failure rates for different structures, as given in Figure 2-3. Since the target nominal reliability is higher than the observed values, sometimes the observed range is increased arbitrarily to specify target levels.

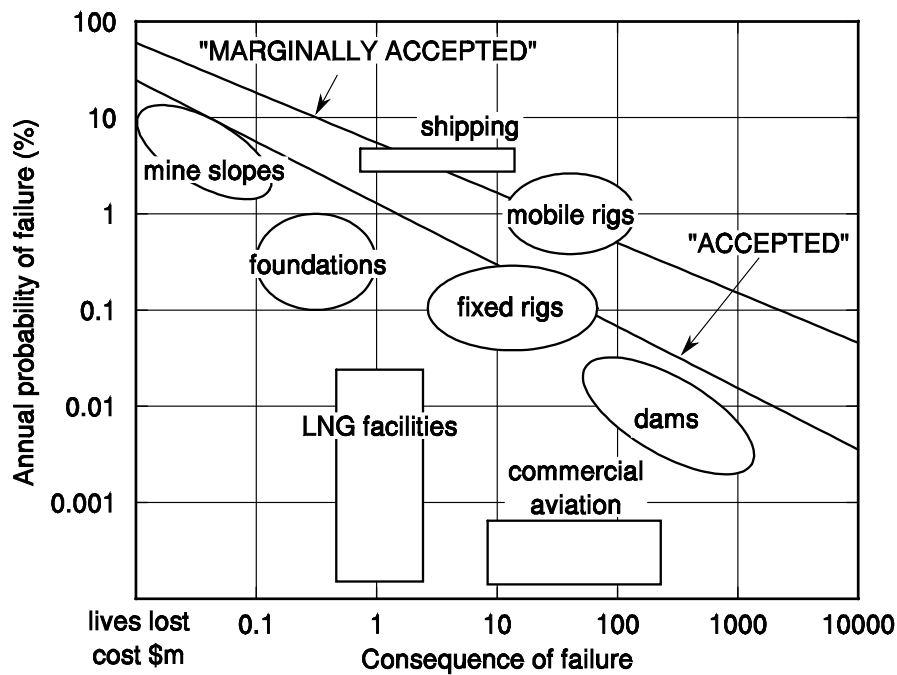


Figure 2-3. Observed failure rates and acceptable limits (from Baecher 1987)

The third method, acceptable failure rates, is essentially equivalent to the second approach. Acceptable rates are determined from the assumption that observed rates are tolerated by society. In general, there is a negative correlation between the acceptable probability of failure and the consequences of failure. Acceptable levels can be plotted as in Figure 2-3 or in F-N charts, in which the frequency or failure rate (F) is plotted on the vertical axis and the consequence (N) is plotted on the horizontal axis. Acceptable limits typically are straight lines in logarithmic plots. Figures 2-4 and 2-5 show two examples of acceptable risk levels for urban development and dam design. In Figure 2-4, three risk zones are defined: acceptable risk, ALARP region, and unacceptable risk. ALARP stands for “as low as reasonably practicable”, and it refers to a zone in which the cost of further reducing the risk is unreasonably high. Figure 2-5 shows tolerable upper and lower bounds according to ANCOLD (1994) and Whitman (1984).

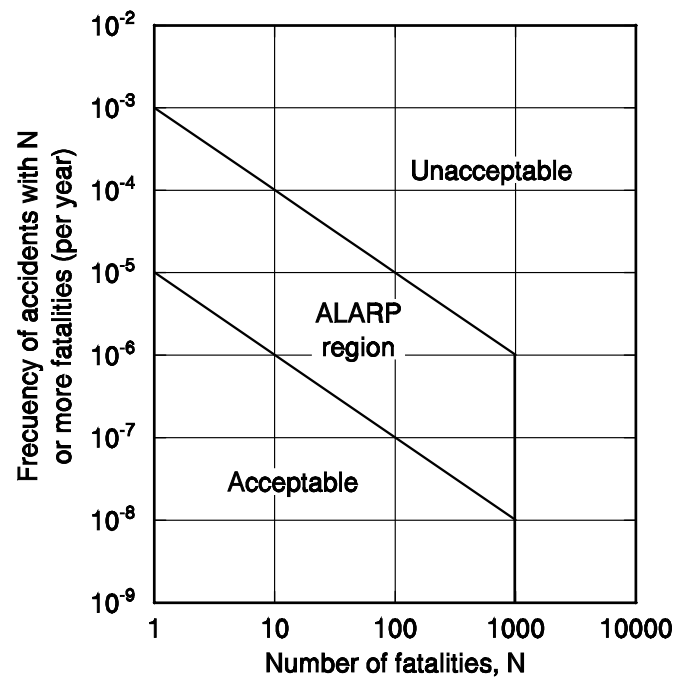


Figure 2-4. ALARP approach (HKGPD 2005)

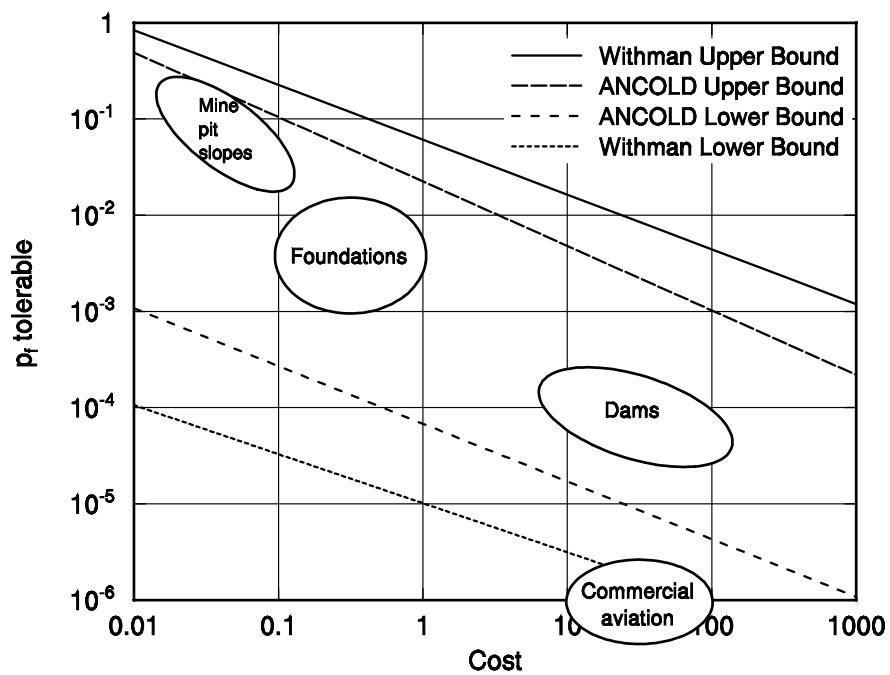


Figure 2-5. Tolerable safety (ANCOLD 1994)

It is interesting to note that acceptable limits are not always directly related to the consequences across all industries. Commercial aviation is an example of an industry with very low failure tolerance. The consequences of dam failures are approximately in the same range of airplane failures. However, the acceptable probabilities are much lower in the latter. In general, acceptable reliability levels are lower for known risks.

The last alternative is to compute the optimum reliability level based on cost-benefit analyses. This approach is very attractive, because it can incorporate failure costs and compare different designs rationally. Unfortunately, the problem is not easy to solve. One of the difficulties is that all the costs and benefits must be evaluated in monetary terms. Initial costs can not be very accurate, because they are estimated before the final design is available. Also, benefits are very difficult to predict even roughly. Engineers should seek help from economists or cost-benefit analysis experts to make sure that the methods and values used are consistent with the objectives of the analysis. Their input can also be very helpful for the selection of failure cost from loss of life and other controversial issues. Another difficulty is that the expected cost of failure must be computed using the true probability of failure, not nominal values (See Chapter 4). Lastly, it is very difficult to compute probabilities of failure when multiple components or multiple failure modes are present. Many foundation systems have redundant elements and can redistribute loads before a failure occurs.

In any case, a target reliability level is necessary to design a structure using any of the current reliability-based methods. Baecher and Christian (2003) indicated that most modern foundation codes contain target reliability indices ranging from 2.0 for non-essential designs with high redundancy to 3.0 for critical designs with high redundancies. In a comprehensive study, Phoon et al. (1995) recommended target reliability indices for foundation design of transmission line structures of 3.2 for ULS

and 2.6 for SLS based on several considerations. Nowak (1995) used a target reliability index of 3.5 for calibration of LRFD equations used in bridge design. His selection process was not discussed in detail, but apparently the number came from the reliability of existing structures.

2.4 Partial Safety Factors

The partial safety factor method is considered an improvement over the global factor of safety, because it can achieve more uniform reliabilities (Meyerhof 1995). When the partial factors are calibrated properly, it is a simplified probabilistic method similar to LRFD, although there are some fundamental differences between the two approaches. In this method, a design is considered satisfactory when:

$$g(x_{d1}, x_{d2} \dots x_{dn}) > 0 \quad (2-13)$$

in which $g(\cdot)$ = performance function and x_{di} = design value of parameter i .

The design value of each parameter is equal to the product of a characteristic value times the partial safety factor, as given below:

$$x_{di} = FS_i \cdot x_{ki} \quad (2-14)$$

in which x_{ki} = characteristic value of parameter i .

If the most unfavorable case occurs for low values of the parameter, the characteristic value is divided by the partial safety factor, as given below:

$$x_{di} = \frac{1}{FS_i} \cdot x_{ki} \quad (2-15)$$

In this form, all target partial safety factors are greater than or equal to one.

In principle, 2^n design points should be checked to determine the least favorable combination of design values, because that is the number of possible combinations for high and low values of each parameter. Fortunately, often it is known whether low or high values control the design. For instance, typically low values of the friction angle are less favorable in foundation engineering. A simple example with two random variables is shown in Figure 2-6.

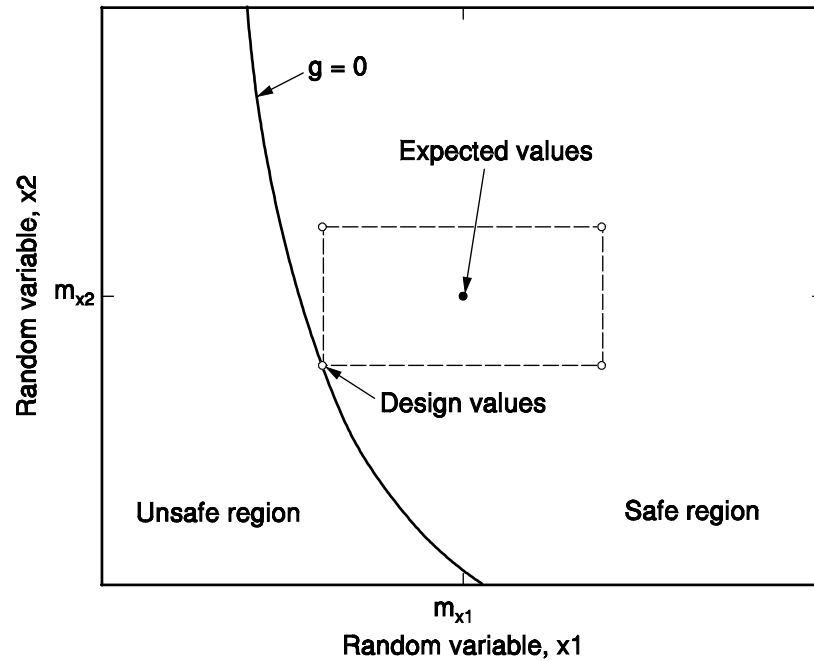


Figure 2-6. Partial safety factors for two random variables

The partial safety factors can be calibrated to achieve a target reliability index using full probabilistic methods or the following simplified approach. If the design parameter is normally distributed, then the design value is:

$$x_{di} = m_{xi} (1 - \alpha_{xi} \beta_T \text{COV}_{xi}) \quad (2-16)$$

in which m_{xi} = mean value of x_i ; α_{xi} = sensitivity coefficient; β_T = target reliability index and COV = coefficient of variation of x_i .

The sensitivity coefficient is similar in concept to the separation coefficient used for LRFD calibration, which is a dimensionless separation of the design value from the mean value necessary to achieve the desired reliability index.

Assuming that the characteristic value of x is equal to the mean value ($x_{ik} = m_{xi}$), and combining Equations 2-14 and 2-16, results in:

$$\text{FS}_i = (1 + \alpha_{xi} \beta_T \text{COV}_{xi}) \quad (2-17)$$

It appears that partial factors can be obtained simply by using Equation 2-17, and every random variable can be treated independently. However, the sensitivity coefficient is not a constant. The same random variable can have different α values depending on the form of the limit state equation and its influence on the result. In other words, it is necessary to use a different partial factor for each random variable and reliability, and for each limit state equation. For example, the partial factor for the undrained strength in a drilled shaft design problem should be different for the side and the tip resistance. As a result, many factors are required to cover all the possible combinations of design equations, target reliabilities, and coefficients of variation. In practice, the sensitivity coefficients are taken as constant for certain problems (see Subsection 2.6.4).

2.5 Load and Resistance Factor Design

The load and resistance factor design (LRFD) approach is a simplification to achieve a target reliability level without the effort required in other probabilistic methods. The basic idea is to modify each term of the limit state equation by using load and resistance factors, as given below and shown in Figure 2-7:

$$\eta F_N = \psi Q_N \quad (2-18)$$

in which η = load factor and ψ = resistance factor.

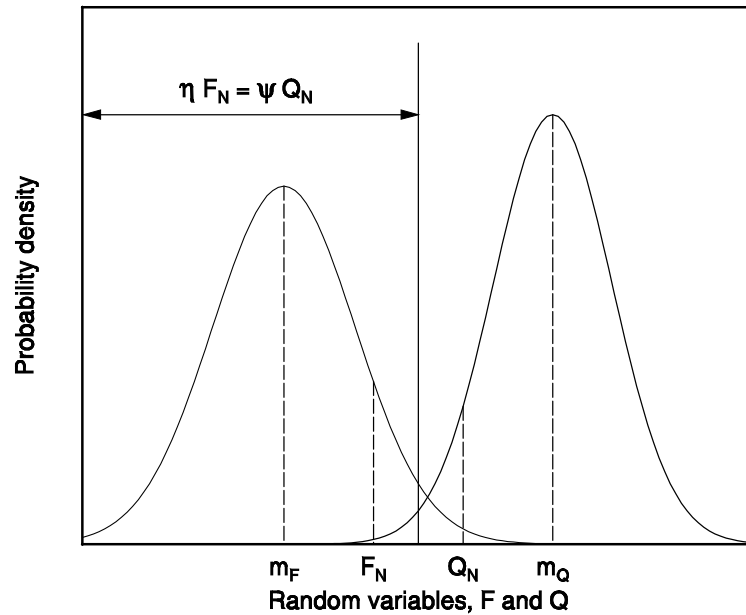


Figure 2-7. LRFD format for normal load and resistance terms

The subscript N in Eq. 2-18 indicates that the load and resistance values are nominal, because in most cases they are not equal to the mean values. When there is only one term on each side of the equation, the LRFD approach is equivalent to the global safety factor method, with $FS = \eta / \psi$.

The typical form of LRFD equations in structural engineering has different load factors for each load type (e.g. live, dead, wind, earthquake) depending on its variability, but only one resistance factor is used. The basis for this format is that loads have much higher variability than man-made materials; relatively uniform reliability can be achieved with only one factor on the resistance side. However, the properties of natural geomaterials typically are more variable than the loads. In many cases, a single equation can not guarantee uniform reliability for all the possible values of the design parameters.

Some authors have proposed LRFD equations with a different factor for each resistance term (Phoon et al. 1995). These equations can achieve higher uniformity of reliability levels, because every component of the resistance can be modified. For example, the capacity of drilled shafts can be computed as the sum of the side and tip resistances and the weight, but each term will have a different contribution to the overall capacity. This format is called multiple load and resistance factor design or MRFD.

2.5.1 Calibration Procedure

There are a number of simple procedures to calibrate LRFD equations when the performance function consists only of a lumped resistance and a lumped load term. For example, the resistance factor, ψ , for a particular global factor of safety is:

$$\psi = \frac{\eta}{FS} \quad (2-19)$$

Using the same simplified form (Eq. 2-18), the factors required for a target reliability index are:

$$\psi = (1 - 0.75\beta_T \text{COV}_Q) m_Q / Q_N \quad (2-20)$$

$$\eta = (1 + 0.75\beta_T \text{COV}_F) m_F / F_N \quad (2-21)$$

If the load and resistance terms have lognormal distributions, the resistance factor is:

$$\psi = \frac{\eta \left[(1 + \text{COV}_F^2) / (1 + \text{COV}_Q^2) \right]^{0.5}}{\exp \left\{ \beta_T \left(\ln \left[(1 + \text{COV}_Q^2) (1 + \text{COV}_F^2) \right] \right)^{0.5} \right\}} \quad (2-22)$$

The derivation of Equations 2-20, 2-21, and 2-22 is given in Appendix A.

A more general calibration procedure for LRFD and MRFD equations is described by Ellingwood et al. (1980) and has been used to calibrate some structural design codes. The method also was used to calibrate the MRFD equations for foundation design of transmission line structures (Phoon et al. 1995). This general approach, which can handle multiple load and resistance terms, consists of the following steps:

- (a) Perform a sensitivity analysis to determine the influence of the variability of each parameter on the resulting reliability.
- (b) Partition the statistical parameter space (means and standard deviations) into smaller subdomains (typically three) for the most significant parameters. Partitions sizes need not be equal.
- (c) Select a set of points inside each subdomain, ideally covering the entire area, and determine an acceptable design for each point.

- (d) Determine the reliability of each design and compute the deviation from the target reliability as below:

$$H(\psi_i, \eta_i) = \sum (\beta_i - \beta_T)^2 \quad \forall i \quad (2-23)$$

in which ψ = resistance factor, η = load factor, β_i = reliability index of the i^{th} design point, β_T = target reliability index, and $\forall i$ = for all i .

- (e) Adjust the load and resistance factors until the objective function, $H(\cdot)$, is minimized.
- (f) Repeat steps d and e for each subdomain.

Once the factors have been determined for each subdomain, the designer can select the applicable equations depending on the statistical properties of the specific design problem. This procedure will be used for MRFD calibration using target safety levels obtained from cost-benefit analysis.

2.5.2 Importance Factors

Some codes and design methods use importance factors to consider empirically the different consequences of failure. In most cases, structures are divided into two or three categories depending on their function. For example, the design loads specified in ASCE 7 (see Subsection 2.6.1) are multiplied by an importance factor that can be less than one for non-critical buildings or greater than one for important ones. This empirical approach has been implemented and accepted by the design community because it is a reasonable modification. The requirement is similar to a higher global safety factor for important structures.

An equivalent adjustment in RBD methods would be to use higher target reliability indices for important structures, but this is rarely done in practice.

García-Pérez et al. (2005) calculated the optimum importance factor for seismic hazards, assuming a function to calculate the initial cost of a structure and a monotonically increasing function to estimate the damage costs as a function of the intensity of the earthquake. The study considered only two categories of structures: ordinary and important. It does not deal with failure costs, because the optimum values are relative to an importance factor of 1.5 at a site distant from the source.

Selection of importance factors is rarely discussed in the literature; apparently, there is no rational basis for their selection other than achieving a higher nominal reliability. Although these modifications are intuitively appropriate, the figures used in some building codes are not consistent, as shown in the following section.

2.6 Building Codes

Contrary to common belief, the purpose of building codes is not only to protect building owners and occupants against inadequate design. They also serve as guidelines for engineers and to protect the public from irresponsible or ignorant owners who disregard safety by trying to reduce initial costs. Typically, building codes only provide minimum safety requirements, because the main concern is to prevent structural and serviceability failures, not economic losses. The task of designing efficient and economic structures is left to the designers. Today, we enjoy relatively low failure rates as a consequence of this conservative building code philosophy. Unfortunately, the same philosophy is responsible for an undetermined number of overdesigned and unnecessary foundations.

In a global cost optimization framework, those oversized foundations should be considered also as failures, because they lead to an unwanted result. Even so, the priority is correct; high initial costs are less serious failures than structural collapses in most cases. However, with a more ambitious attitude, building codes could protect owners and society against economic failures as well.

Some modern building codes include simplified probabilistic approaches to achieve target nominal reliabilities, recognizing the need to include uncertainty in the design. It is necessary to use simple techniques, because full probabilistic methods would be too complex and time-consuming for the design of conventional projects. Unfortunately, designing structures for a fixed target reliability does not guarantee minimum life-cycle costs. Building codes should integrate methods to achieve acceptable, near-optimum safety levels, considering the variability in the design parameters and the consequences of failure. The probabilistic approaches used in different building codes are discussed next.

2.6.1 ASCE 7

The Minimum Design Loads for Buildings and Other Structures Standard (ASCE 7) is not a comprehensive building code. It is a specification to determine conventional loads acting on structures, including gravitational, earth, flood, wind, rain, ice, snow, and earthquake loads. Chapter 2 in ASCE 7 contains load combinations that should be used for the design of structures, components, and foundations. Those load combinations incorporate load factors for structural design using either LRFD or ASD. Therefore, the nominal values and factors specified in this standard must be used with compatible design methods and conventional material properties.

Most of the statistics and factors in ASCE 7 are based on a report prepared for the National Bureau of Standards (Ellingwood et al. 1980). In that report, the authors derived LRFD equations based on the bias, variance, and expected values of different loads and typical structural members. Subsequent studies have enhanced the results, incorporating new statistical data and building methods. However, the original approach, in which load factors are selected from a specific target reliability, is still used.

It is important to note that factored loads are not used commonly in foundation design for several reasons. First, load factors were calibrated according to their probability distributions and the variations of structural materials, not for the typical variability of soil parameters. By simple inspection of Equation 2-22, it is evident that load factors are a function of the resistance factor and its variability. Load factors can be specified for structural design without direct mention of resistance factors, because the variability and bias of traditional materials is small relative to the variability of the loads. In the case of foundation design, where soil strength can be highly variable, LRFD equations must be calibrated including both the load and resistance terms.

Consequences of failure are considered in ASCE 7 by means of empirical importance factors. Each structure must be classified in one of the four categories available. Then, an importance factor is prescribed for each category to adjust the nominal flood, wind, snow, earthquake, and ice loads. The description of each building category is presented in Table 2-5.

The reference load is defined for category II structures (importance factor = 1.0). Low-hazard structures (category I) have importance factors less than one, while important structures have factors greater than one. Importance factors range from 0.77 to 1.15

for wind design and from 1.0 to 1.5 for earthquake design. Although the factors are empirical, they are consistent with the variability and cost-of-failure principle of decision theory. Unfortunately, there is no rational support for these values; it is common to find large discrepancies between different building codes.

Table 2-5. Occupancy categories according to ASCE 7

Nature of occupancy	Occupancy category
Buildings and other structures that represent a low hazard to human life in the event of failure (agricultural and storage facilities)	I
All buildings and other structures except those listed in Occupancy Categories I, III, and IV	II
Buildings and other structures that represent a substantial hazard to human life in the event of failure (schools, jails, more than 300 people). Buildings and other structures, not included in Occupancy Category IV, with potential to cause a substantial economic impact and/or mass disruption of day-to-day civilian life in the event of failure (power generator stations, water treatment, sewage)	III
Buildings and other structures designated as essential facilities (hospitals with emergency facilities, fire, ambulances, police, shelters, water for fire suppression, aviation control towers, ancillary structures). Buildings and other structures containing highly toxic substances where the quantity of the material exceeds a threshold established by the authority.	IV

Importance factors for wind and seismic loads are given in Tables 2-6 and 2-7.

Table 2-6. Importance factors for wind loads (ASCE 7)

Occupancy category	Non-hurricane prone regions and hurricane prone regions with $V = 85\text{-}100$ mph (38-45 m/s) and Alaska	Hurricane prone regions with $V > 100$ mph (45 m/s)
I	0.87	0.77
II	1.00	1.00
III	1.15	1.15
IV	1.15	1.15

Note: V = basic wind speed

Table 2-7. Importance factors for spectral seismic design (ASCE 7)

Occupancy category	Importance factor, I
I or II	1.0
III	1.25
IV	1.5

2.6.2 International Building Code

The International Building Code or IBC (ICC 2009) was developed in the United States by the International Code Council as a successor of three older codes: the BOCA National Building Code (BOCA 1993), the Uniform Building Code (ICBO 1997), and the Standard Building Code (SBCCI 1999). The first edition of the IBC was published in 2000 and contains elements of the three legacy codes. Most of the structural design specifications use the LRFD approach, but Chapter 18, Soils and Foundations, still uses the ASD approach. Naturally, the allowable stresses of

geomaterials given for foundation design are very conservative, but higher values are permitted if a site investigation justifies the increase.

The code defines several load combinations that must be considered for the design of each structure or component. The load combinations for foundation design (ASD) are the following:

$$\begin{aligned} &D \\ &D + L \\ &D + L + (L_r \text{ or } S \text{ or } R) \\ &D + (W \text{ or } 0.7 E) + L + (L_r \text{ or } S \text{ or } R) \\ &0.6 D + W \\ &0.6 D + 0.7 E \end{aligned} \tag{2-24}$$

in which D , L , L_r , S , R , W , and E = dead, live, live roof, snow, rain, wind, and earthquake loads, respectively.

Some of the loads used in Equation 2-24 can be modified by the importance factors specified in ASCE 7. Seismic, wind, and snow loads can be increased or reduced according to the building categories defined by the IBC. In this way, the consequences of failure are explicitly considered in foundation design using ASD.

The IBC is also the basis for the Building Code of New York State (2007). Most of the structural requirements, loads, and design methods are adopted from the IBC. There are only minor modifications made by the State Fire Prevention and Building Code Council.

In summary, the IBC is an attempt to integrate a simplified probabilistic approach calibrated for typical statistical properties of loads and resistances. This code deserves

credit for unifying previous codes and serving as a transition towards a reliability-based approach, but there are three key issues that must be addressed.

- (a) Calibration of LRFD equations is not available to the public.
- (b) Foundation design still uses the ASD approach, but the loads are affected by importance factors.
- (c) Importance factors do not have a rational basis and are inconsistent with other building codes.

2.6.3 National Building Code of Canada

The National Building Code of Canada (NRC 1995) was one of the first codes to implement a simplified probabilistic approach. The 1985 edition of the code specified ASD for foundations, but there was a provision in the code for LRFD, called limit state design (LSD), of structural elements. Only one importance factor is considered for earthquake design and is included in LSD load combination equations; therefore, the nominal loads are not affected by the consequences of failure. The specified ASD method can not consider costs, because the load combinations do not mention importance factors. A sample LSD load combination is shown in Eq. 2-25, and the importance factor values are shown in Table 2-8.

$$1.25 D + I lcf [1.5 L + 1.5 Q + 1.25 T] \quad (2-25)$$

in which Q = wind or earthquake load, T = temperature load, I = importance factor, and lcf = load combination factor (1.0 for one acting load, 0.7 for two loads, 0.6 for all loads).

Table 2-8. Importance factor for earthquake design (NBCC 1985)

Occupancy category	Importance factor
Post-disaster buildings and schools	1.3
All other buildings	1.0

Similar to the IBC, the NBCC does not have a rational method for the selection of importance factors. In contrast, the importance factor for seismic design can be either 1.0 or 1.3, while in the IBC, the same factor ranges from 1.0 to 1.5. Another significant difference is that the NBCC does not modify acceptance criteria for ASD based on the importance of the building.

2.6.4 Eurocode

The Eurocode is a pan-European standard composed of a head document (EN 1990, Basis for Structural Design) and nine volumes for different construction materials and aspects of building design. It is a mandatory code for European public works since March 2010, replacing national building codes in all member states. However, each country is expected to issue a national annex.

Eurocode 7 (EN 1997 Geotechnical Design) (CEN 2001) contains specifications for typical geotechnical design problems. The partial safety factor design approach, described in Section 2.4, is used for both structural and geotechnical design.

In this code, the target partial factors are typically obtained with Equation 2-17. The target reliability index and the coefficients of variation of the significant parameters can be specified by the designer for each project. However, the sensitivity coefficients, which are given in tables, must be determined with probabilistic analyses.

When lumped load and resistance terms are used, another simplification is used in the code. The sensitivity coefficients are given in tables only for two types of variables: dominant and non-dominant cases (Table 2-9).

Table 2-9. Standardized α values (from CEN 2001)

	Load	Resistance
Dominant variables	0.70	0.80
All other variables	0.28	0.32

The code specifies target reliability indices for ultimate, serviceability, and fatigue limit states according to three structure categories, as defined in Table 2-10, depending on their importance and the consequences of failure. Each consequence class (CC1, CC2, and CC3) is associated with a reliability class (RC1, RC2, and RC3). The recommended minimum reliability index for each class is shown in Table 2-11.

The target reliability indices for the reference category (CC2) are shown in Table 2-12. Reliability for other reliability classes in SLS design can be achieved by using the multiplication factor for actions, K_{FI} , shown in Table 2-13 for the calculation of partial safety factors.

In addition to the typical structural categories, Eurocode 7 defines three geotechnical categories according to the importance and complexity of the project.

Category 1 (GC1), which is defined as low geotechnical hazard, includes only small and relatively simple structures with negligible risk of property or life loss because of ground or load conditions. Empirical methods are acceptable to ensure adequate reliability.

Table 2-10. Consequence classes in Eurocode (EN 1990, Annex B)

Consequence class	Description	Examples of buildings and civil engineering works
CC3	High consequence for loss of human life, or economic, social or environmental consequences very great.	Grandstands, public buildings where consequences of failure are high.
CC2	Medium consequence for loss of human life; economic, social or environmental consequences considerable.	Residential and office buildings, public buildings where consequences of failure are medium.
CC1	Low consequence for loss of human life, and economic, social or environmental consequences small or negligible.	Agricultural buildings where people do not normally enter (e.g. storage buildings), greenhouses.

Table 2-11. Recommended minimum values for reliability index of ultimate limit states (EN 1990)

Reliability class	Minimum values for β (ULS)	
	1-year period	50-year period
RC3	5.2	4.3
RC2	4.7	3.8
RC1	4.2	3.3

Table 2-12. Target reliability index for class CC2 structural members (EN 1990)

Limit state	Target reliability index	
	1-year period	50-year period
Ultimate	4.7	3.8
Fatigue	-	1.5 to 3.8
Serviceability	2.9	1.5

Table 2-13. Multiplication factor for actions (EN 1990)

K_{FI} factor for actions	Reliability class		
	RC1	RC2	RC3
K_{FI}	0.9	1.0	1.1

Category 2 (GC2) or moderate geotechnical hazards are conventional structures with no abnormal risks or conditions. A quantitative geotechnical analysis must be done by a qualified person, normally a civil engineer with geotechnical experience.

Category 3 (GC3) or high geotechnical hazard includes projects that do not fall in categories 1 or 2. It includes large or unusual structures, difficult ground conditions, or high seismicity areas. An experienced geotechnical specialist must approve the design.

Interestingly, this category specification not only increases the nominal reliability but also the true reliability. In principle, an extended site investigation, sophisticated analyses, and highly experienced personnel not only improve the quality of the predictions but also decrease the possibility of human errors.

In the author's opinion, the partial safety factor approach has more limitations than LRFD as a simplified probabilistic analysis, but the geotechnical categories used in

Eurocode 7 offer an interesting approach to cost optimization. For other reviews of the approach used in Eurocode, see Orr and Breyse (2008) or Vrouwenvelder (2008).

2.6.5 AASHTO

Current AASHTO specifications for bridge design (including foundations) use the LRFD approach. According to Paikowsky (2004), the specified factors were calibrated using a combination of reliability theory, reliability implied in ASD, and engineering judgment. The theory behind LRFD usage and calibration was summarized in the previous section.

The specific calibration procedures used in AASHTO LRFD Bridge Design Specifications (2004) are consistent with general reliability theory. Barker and Puckett (2007) discuss the theory employed for the code. A target reliability index of 3.5 was considered throughout the code for the structural design. For the foundations, values from 3.5 to 2.0 were used. The consequences of failure are not considered explicitly in the calibration procedure; however, the code allows the use of load modification factors to account for ductility, redundancy, and importance. Factored loads are modified according to Eq. 2-26.

$$\sum \kappa_i \eta_i F_i \leq \psi Q \quad (2-26)$$

$$\kappa_i = \kappa_D \kappa_R \kappa_I$$

in which κ_D = ductility factor, κ_R = redundancy factor, and κ_I = importance factor

Ductility, as mentioned previously, can decrease the cost of failure by impeding a sudden catastrophic event. Typically, a ductile element can deform enough to alert the

users of a facility that something is not working as intended. Further inspection by experts can reveal the cause of excessive deformations and prevent catastrophic failures. Also, ductile elements in redundant structures allow redistribution of loads to other elements. The code allows the following values:

$\kappa_D \geq 1.05$ for non-ductile components and connections

$\kappa_D = 1.00$ for conventional designs and details complying with specifications

$\kappa_D \geq 0.95$ for components and connections for which additional ductility-enhancing measures have been specified beyond those required

Redundancy is the term used to describe the possibility that loads take different paths from the initial point to the supporting elements. The canonical example of a non-redundant bridge structure is an isostatic truss. If one element fails, the loads can not take a different path to the foundation, and collapse is inevitable. In foundation engineering, pile foundations typically are redundant; if the capacity of one pile is less than the design value, the excess load is redistributed to adjacent piles as long as there is an adequate safety margin. The redundancy factors specified by AASHTO are:

$\kappa_R \geq 1.05$ for non-redundant members

$\kappa_R = 1.00$ for conventional levels of redundancy

$\kappa_R \geq 0.95$ for exceptional levels of redundancy

Finally, the effect of importance is also considered with a single factor. The reason to increase safety in important structures should be clear by now. The importance factors are:

$\kappa_I \geq 1.05$ for a bridge of operational importance

$\kappa_I = 1.00$ for typical bridges

$\kappa_I \geq 0.95$ for relatively less important bridges

According to Equation 2-26, the factored load of a redundant, ductile element in a less important bridge should be multiplied by $0.95^3 = 0.86$. However, the minimum global modification factor, κ_i , allowed by the code is 0.95.

Unlike other codes, AASHTO includes an additional importance consideration for seismic design. This extra modification is a reduction in seismic loads obtained from elastic analyses. The logic for this reduction is that, during an earthquake, real elements will have inelastic deformations that reduce its stiffness and, ultimately, the seismic load. However, the response modification factors also depend on the importance category. The motivation to include structural importance in the selection of reduction factors is understandable, but it would make more sense to adjust the design equations only one time.

Table 2-14 describes the three bridge categories used by AASHTO for seismic design. The calculated load effects must be divided by the factors shown in

Table 2-15.

Table 2-14. Classification of bridges according to importance

Importance category	Description
Critical bridges	Must remain open to all traffic after the design earthquake (475-year return period) and open to emergency vehicles after a large earthquake (2500-year return period)
Essential bridges	Must be open to emergency vehicles after the design earthquake
Other bridges	May be closed for repair after a large earthquake

Table 2-15. Response modification factors for substructures (AASHTO 2004)

Substructure	Other	Essential	Critical
Wall-type piers	2.0	1.5	1.5
Reinforced concrete pile bents			
Vertical piles only	3.0	2.0	1.5
One or more batter piles	2.0	1.5	1.5
Single columns	3.0	2.0	1.5
Steel or composite steel and concrete pile bents			
Vertical piles only	5.0	3.5	1.5
One or more batter piles	3.0	2.0	1.5
Multiple column bents	5.0	3.5	1.5

2.7 Summary

This overview of available design methods illustrated how cost is included in the selection of target safety levels. Classic and modern structural design methods were described briefly without addressing detailed design philosophy or historic development. However, the reader can refer to the recommended literature for detailed procedures, specifications, and examples.

The consequences of failure have been considered either intuitively or empirically even before modern soil mechanics appeared. All of the methods include some way to consider the potential cost of failure, but they do not attempt to achieve optimal designs. In general, engineers must adjust the design empirically to minimize initial costs while satisfying safety requirements. None of the available design methods has rational arguments to modify the design based on ductility, redundancy, or importance.

While research in probabilistic design methods has increased in recent years, building code implementation moves at a slower pace for several reasons. Some codes, especially for geotechnical design, still use ASD or the global safety factor approach as recommended many decades ago. The transition from ASD to LRFD for geotechnical design has faced substantial opposition by some practitioners. One of the arguments against LRFD is the loss of flexibility to adjust resistance factors based on the engineer's judgment and experience. If the load and resistance factors are fixed, regardless of the quality of the geotechnical investigation, local knowledge, initial cost, and the consequences of failure, the final design using LRFD could be less adequate than a traditional design performed by a competent, experienced engineer.

Building codes have served traditionally to specify minimum safety and serviceability requirements, but they do not consider overdesign as a design failure. For practical purposes, most of the design constraints are derived from conservative assumptions, because the consequences of failure could be much more serious than an economic failure. In the past, target safety levels had to be more conservative, because the uncertainty in the performance function was larger.

Today, because of modern exploration methods, improved analyses, and more accurate statistical properties, the variability of structural performance has decreased. But the calibration methods are based on the reliability implied in traditional methods, preserving the old excessive safety levels. While this situation is undesirable, it seems to be the best option available in the absence of a rational framework.

Some authors have stated that the selection of target safety from cost-benefit analysis is the best approach to this problem, but it is still too complex. If the selection of safety levels is done with a rational approach, building codes will be able to indicate

not only minimum requirements, but near-optimum designs for the particular location and resources available. Then the design philosophy behind building codes can shift from failure prevention to cost optimization.

CHAPTER 3

METHODS FOR STRUCTURAL OPTIMIZATION

3.1 Introduction

The drawbacks of available design methods, described in the previous chapter, have been studied broadly. Traditionally, cost minimization and selection of safety levels are addressed independently for practical reasons but in reality they are two aspects of the same problem, which is to obtain the best possible design with the available information. Regarding the former aspect, structural optimization has been used successfully to solve specific problems. The goal of structural optimization is to obtain a design that produces the minimum or maximum value of an objective function while satisfying a set of constraints. This chapter presents a summary of the optimization methods available that have been applied to structural and geotechnical engineering.

Most optimization methods are algorithms that can be classified as gradient-based or heuristic. Initial developments in structural optimization did not include reliability considerations but the majority of recent studies do (Feng and Moses 1986, Ang and De Leon 1997, Sarma and Adeli 1998, Frangopol and Corotis 1996, Frangopol and Maute 2003, Wen and Kang 2001a,b). One of the original applications was to minimize the weight or size of a structural member while satisfying deterministic safety requirements. A recent application of deterministic cost optimization in foundation design was given by Wang and Kulhawy (2008a). Soon after the development of RBD, researchers applied reliability constraints in the optimization process, calling it reliability-based design optimization (RBDO). Today, RBDO is a very active research topic with many of variations in the algorithms, objective

functions, and constraints. When the formulation considers costs over the design life of the structure, the process is called life-cycle cost optimization (LCCO).

Cost optimization methods presented in Chapter 5 can be considered a particular case of RBDO in which: (a) the objective function is the life-cycle cost of a structure and (b) there are no constraints. Some of the new approaches for RBDO are presented in this chapter, noting their advantages and limitations. This literature review includes several types of optimization techniques, even if they do not have cost objectives because they can be modified easily to consider life-cycle cost as their objective.

3.2 Reliability-Based Design Optimization

There have been many recent papers on LCCO and RBDO (Nakanishi and Nakayashu 2002, Castillo et al. 2004, Abdelatif 2007, Liang et al. 2007, Aoues and Chateauneuf 2008, Babu and Basha 2008, Wang and Kulhawy 2008a, Zhang et al. 2011). However, almost all of these studies use optimization techniques with reliability constraints, as given below:

$$\begin{cases} \text{minimize } C_1(d) \\ \text{subject to } \beta_i \geq \beta_{Ti} \text{ for } i = 1 \text{ to } n \\ \text{and } d_L < d < d_U \end{cases} \quad (3-1)$$

in which C_1 = initial cost (objective function), d = vector of design variables, β_i = reliability of i^{th} failure mode, β_{Ti} = target reliability of i^{th} failure mode, n = number of failure modes, and d_U and d_L = vectors of upper and lower bounds for the design variables.

The same RBDO problem can be expressed in different ways by changing the constraints. When constraints are expressed in terms of the reliability index, as shown in Equation 3-1, the method is said to use the reliability index approach (RIA). When the safety requirements are expressed in terms of the performance, the method is said to use the performance measure approach (PMA), as shown below:

$$\begin{cases} \text{minimize } C_I(d) \\ \text{subject to } P(g_i < 0) < \Phi(-\beta_{Ti}) \text{ for } i = 1 \text{ to } n \\ \text{and } d_L < d < d_U \end{cases} \quad (3-2)$$

Two studies were conducted to compare the efficiency of the methods (Tu et al. 1999, Youn and Choi 2004). Both concluded that PMA is more stable but the efficiency depends on the characteristics of the problem. In any case, if reliability-based methods and explicit performance functions are used, the problem can be solved with a relatively small computational effort, and therefore efficiency is not a relevant issue.

In a global reliability life-cycle cost optimization framework, the model has no reliability constraints. The framework proposed herein is different from other studies, because it determines the optimum reliability levels, including the expected costs of failure, as given below:

$$\text{minimize } C_T(x) = C_I(x) + C_F(x) p_f \quad (3-3)$$

in which C_I = initial cost, C_F = cost of failure, and p_f = probability of failure.

The benefits and caveats of the cost approach will be discussed in the next section, but it is evident that no explicit safety requirements are needed. The optimum design point

is determined by the rational balance between initial costs and safety, not by fixed reliability constraints.

It is important to distinguish between the two approaches, because the final objective is quite different. Wang and Kulhawy (2008a) used the name reliability-based economic optimization to describe an application of RBDO to shallow foundation design. The example presented is equivalent to a cost-objective function with simplified reliability constraints. Studies that also consider life-cycle costs and unconstrained optimization include Turkstra (1967), Rosenblueth (1986), Kanda and Ellingwood (1991), Gasser and Schueller (1997). Unfortunately, the name life-cycle cost optimization (LCCO) has been associated with the first type of economic analysis. It is proposed herein to use the name unconstrained reliability-based economic optimization (URBEO) to identify the second approach. This name implies that the formulation has no arbitrary constraints, the probability of failure is estimated with reliability-based methods, and the objective is an economic measure that can be either cost or benefit.

3.2.1 Types of Optimization Techniques

Optimization refers to a computational problem whose objective is to select the best alternative from all the possible solutions. In mathematics, the solution of an optimization problem is the set of independent variables that minimize or maximize a real-valued objective function.

In most cases, optimization problems have a single objective function, but there are methods to ponder multiple objectives. Regarding the form of the objective function and the constraints, problems can be linear or nonlinear. In a linear problem, the

objective function and the constraints are linear combinations of the design variables. When the design variables can take only integer values the solution process is called integer programming. Within the nonlinear optimization problems, there are convex and non-convex conditions. Convex optimization requires that the objective function and constraints be convex. If a problem is convex, several theories and efficient algorithms can be used to find the solution. Also, if the function has a minimum, the solution is unique.

There are a number of different algorithms for nonlinear problems with different requirements and efficiencies. In many cases, it is necessary to compute the gradient of the objective function or even the second partial derivatives to apply a technique.

Other groups of methods called heuristics use empirical algorithms to improve designs without the requirements of gradient-based methods. However, heuristic methods can not guarantee mathematically that they have found an optimum solution. Common heuristic methods include genetic algorithms, evolution strategies, hill climbing, and simulated annealing. Since reliability analysis is an optimization problem, heuristic methods can also be used to solve reliability problems (Gavin and Xue 2009).

In general, gradient-based methods are preferred over heuristics, because they require less computational effort and are easier to implement. However, for some difficult optimization problems, heuristics are the only option. Some applications of heuristic methods in geotechnical optimization problems are given by Goh (1999) and Cui and Sheng (2005).

Since the design optimization problem stated in Equation 3-3 is an unconstrained, nonlinear, continuous, real-valued function, gradient-based methods are sufficient to find a solution. The problem may not be convex depending on the cost and limit state

functions. Therefore, a general nonlinear programming algorithm must be used for URBE0. Also, one of the objectives herein is to present methods that can be used with simple, widely available resources.

Probably the most widely available nonlinear optimization engine is an Excel Add-in called Solver. The program is a very efficient implementation of the proprietary generalized reduced gradient (GRG2) algorithm that has been used and tested since its introduction in 1991 (Fylstra et al. 1998). In fact, Solver also uses the simplex method for linear problems and the branch and bound for integer programming. It has been noted that the use of optimization algorithms is not as simple as some people might think. The flexibility of modern spreadsheets makes it easy to create discontinuous functions or even non-numeric results that can not be solved with gradient-based methods. Learning how to obtain meaningful results from optimization software, including Excel Solver, usually takes more time than other operations, because there are a number of options and default settings that may affect the process. A useful general guide to use Excel Solver for cost optimization is given in Appendix B.

3.2.2 Performance Functions

Performance functions can be divided into two groups according to their form: explicit and implicit. Explicit functions are algebraic equations used to indicate the state of a structure, such as the safety factor or the safety margin. In ordinary foundation design problems, the majority of performance functions are explicit.

Other more sophisticated analyses may use implicit functions such as finite element or finite difference methods. Implicit functions are computationally more demanding and are not suitable to use within other numerical optimization algorithms, because

additional iteration loops are required. One exception to this generalization is linear programming since it does not involve iterative solutions. Unfortunately, very few problems in geotechnical engineering are linear.

The traditional limit states described by performance functions are ultimate limit states (ULS) and serviceability limit states (SLS). The difference between these two states is essentially the consequence of failure. The definition of each state is somewhat arbitrary, but typically ultimate failures refer to sudden, catastrophic events while serviceability refers to excessive deformations. For optimization purposes, both cases can be treated in the same way. If the formulation includes reliability constraints, safety requirements for ULS should be higher than for SLS, because the consequences of failure are more expensive. If the problem has no constraints, as in URBEQ, the difference between the two cases will be only the cost of failure. Any failure can be defined as the transition of a structure from a desirable to an undesirable state with an associated economic loss.

3.3 Objective Function

Several different objective functions have been used for deterministic and RBD optimization problems. The most common objectives are cost, benefits, utility, area, volume, and weight. A less common approach is to use the reliability as objective after setting an initial cost constraint. Other studies consider multiple objective functions. Marler and Arora (2004) prepared a survey of available multi-objective RBDO criteria. However, when multiple objectives are used, it is necessary to include an additional criterion to define optimum conditions. Most authors agree that a cost-benefit analysis is necessary to define optimum safety levels rationally (Phoon et al.

2003, Paikowsky 2004, Sanchez-Silva and Rosowsky 2008). In other words, a rational framework should be based on decision theory, where the optimum option has the maximum expected benefits.

Cost-benefit analysis is defined as a formal technique to guide the decision process when two or more alternative projects are considered. According to the Pareto improvement criterion (Layard and Glaister 1994), a change (e.g. a construction project) should be approved only if at least one individual is better off and nobody is worse. This is a very strict restraint because, in most cases, a small group of people will be affected by civil infrastructure projects. Economists often make decisions according to the Kaldor-Hicks criterion (Layard and Glaister 1994), in which a decision is efficient (acceptable) if the winners can compensate the losers even if they don't do it. In other words, the total societal benefits exceed the losses. Typically, it is very difficult to value all the benefits that a project brings. In many projects there are unintended benefits or costs to some users that can not be considered before construction. Still, it is valid to compare the costs of different projects that will bring roughly the same benefits. Economists call this procedure a cost-effective analysis (Layard and Glaister 1994). The term used in structural engineering is cost minimization, which is equivalent to a cost-effective analysis.

Turkstra (1967) was one of the first authors in structural engineering to propose an objective cost function for design optimization. He proposed the minimization of a function that included the expected cost of failure. Later, Rosenblueth and Mendoza (1971) presented the following net-benefit function that had to be maximized:

$$Z = V - C_I - C_F p_f \quad (3-4)$$

in which V = present value of benefits, C_I = initial cost, C_F = cost of failure, and p_f = probability of failure.

They also noted that the replacement policy affects the objective function and the solution. Two policies are mentioned in their paper: repair after failure and destroy after failure. Another common replacement policy would be: repair after failure but destroy (decommission) after end of design life. There are other time-dependent factors that may influence the objective function. However, the replacement policy is not considered in traditional, time-invariant approaches.

The cost minimization and benefit maximization problems (Equations 3-3 and 3-4, respectively) are equivalent when benefits are constant. In structural design, there are some cases when the design may affect the benefits. For example, increasing the diameter of internal columns reduces the usable space inside a building. In foundation engineering, it is reasonable to assume that the benefits are independent of the final design. Therefore, using cost as the objective in time-invariant optimization is a reasonable approach.

The life-cycle costs of civil infrastructure belong to one of the following categories:

- Initial / Construction
- Operation / Inspection / Maintenance
- Decommission
- Failure (ULS or SLS)

Within the failure category, we can distinguish injuries, fatalities, damage to contents, damage to the structure itself, damage to adjacent structures, environmental impact, loss of function, loss of productive time, and reputation damage.

This list is not comprehensive, because some special structures may have extraordinary requirements or failure modes. One example of special structures is the failure of a historic building. The direct economic impact of such a loss may include loss of revenue from tourism. Other losses related to cultural issues are certainly more complex to quantify. In those cases, engineers clearly must fall back on other disciplines to establish reasonable values.

Note that all the quantities in cost-benefit analyses must be corrected for time variations. Typically the costs are expressed in present value, assuming a constant discount rate. For this reason, decommission costs, which occur at the end of the life of a structure, are often disregarded in the analysis.

3.3.1 Initial Costs

The first term in the objective function is the initial or construction cost, which is the amount, in monetary terms, required to complete the project. Initial costs comprise all the typical items included in a project bid, such as materials, labor, equipment, indirect costs, and other.

The actual initial cost of a structure can be known accurately only after the end of construction because of possible changes in prices or conditions, delays, failures, etc. Every person in the construction industry knows from experience of the variability in construction costs. One study found that the real cost of transportation infrastructure projects is on average 28% higher than the initial estimates (Flyvbjerg et al. 2002). The same study concludes that cost underestimation is a global phenomenon, although it appears to be more pronounced in developing countries.

For obvious reasons, the design optimization must be done before the construction phase of the project, using only cost estimates. Even as initial costs are not deterministic, the expected cost of failure is much more uncertain in a life-cycle context. Consequently, the assumption of deterministic initial costs has a small influence on the final design. There are other assumptions that have more serious consequences in the design. For example, the market cost of some items may not represent their real cost. For cost-benefit analysis, it is necessary to use shadow prices to optimize an objective function, otherwise the results will be incorrect. In simple terms, the shadow price of an item is the change in the objective function when a constraint on the availability of the item is relaxed by one unit.

Often, market prices are good approximations for shadow prices, but economists know that this is not always the case. Some of the factors that can affect market prices are monopolies, indirect taxes, and unemployment. Other difficulties arise when non-market items must be valued, such as reputation, time, consumer satisfaction, etc. It is recommended that designers work together with experts in cost-benefit analysis to select the adequate initial and failure costs.

3.3.2 Operation, Inspection, and Maintenance

The design of some structures may affect the operation and maintenance cost. In theory, the operation and maintenance costs can be included in the design optimization framework proposed herein. However, foundations rarely receive maintenance during their design life. For that reason, only initial and failure costs are considered in Chapter 5.

In contrast to foundations, other type of geotechnical structures, such as retaining walls or slopes may receive regular inspections and be repaired if necessary. It is possible to determine the optimum number of inspections using time-variant formulations (Frangopol et al. 1997). Kong and Frangopol (2004) proposed a cost function that includes the effect of maintenance interventions on system reliability. Other models can estimate optimum inspection and maintenance intervals (Rackwitz et al. 2005). In general, these optimization approaches show that inspection and maintenance must be planned in the design phase to achieve the minimum life-cycle cost. Certainly, time is a variable that affects the optimum reliability level, and time-invariant simplifications may be inadequate in some cases. However, it is necessary to establish a solid framework for time-invariant models before dealing with more complex problems.

3.3.3 Failure Costs

The cost of failure is probably the most difficult and controversial item of the objective function. In ULS type of failures, the potential losses include human lives, injuries, reputation, damage to the environment, etc. Kanda and Shah (1997) examined sources of failure cost. Some people believe that it is immoral to assign prices to human lives since life is invaluable. Others argue that all lives should have the same value everywhere, regardless of the local economy. The problem arises from misunderstanding of the concept. In reality, the “cost of a human life” is the maximum amount of resources that should be invested to save a statistical life. If the investment exceeds the limit, it means that those resources could be used more effectively elsewhere to save the same statistical life. In other words, it is not the cost of a life; it is the cost to save one. The same idea can be applied to the other items.

Several researchers in engineering and economics have addressed the cost of loss of life and limb (Lind 1994, Rosen 1994, Rackwitz 2002, Sanchez-Silva and Rackwitz 2004). The methods to value loss of human life can be classified as behavioral and non-behavioral. Different approaches yield different estimates of the value of a statistical life (VoSL).

It is argued that for social cost-benefit analyses, the VoSL has little influence since the probability of death is already very small (Jongejan et al. 2005). There are other scenarios where this may not be true. Construction and mining are among the most dangerous activities, so the selected VoSL should have large influence on structural safety and safety measures in these activities. A few methods can be applied without having to assign a value. For example, the optimum target reliability may be that for which the cost of saving an extra life is minimum. However, the use of these methods in a complete societal cost-benefit analysis is not possible because the cost of lives must be quantified in monetary terms. Regardless of the method used, most studies agree on an approximate range that is a function of economic welfare and other social indicators. Table 3-1 presents a comparison of VoSL obtained with different methods.

Table 3-1. Comparison of statistical life values in developed countries (adapted from Jongejan et al. 2005)

Valuation method		VoSL (2005 USD)
Behavioral	stated preference	-
	revealed preference	0 - 10^{10}
Non-behavioral	macro-economic valuation	0.6×10^6
	life quality index	1 - 5×10^6

In this context, revealed preference methods are based on the assumption that economic behavior reflects the value assigned to human life.

Assessment of environmental costs is another controversial task that has been studied recently by a number of authors. Dasgupta and Maler (1994) presented an extensive discussion of environmental issues, such as selection of shadow prices for environmental resources, the market failure to reflect environmental costs, and the dilemma known as “tragedy of the commons”, where unrestricted access to a finite communal resource leads to its depletion. They pointed out that there is a close link between poverty and environmental degradation.

Other authors have presented models to evaluate the life-cycle environmental cost of construction. Treloar et al. (2000) proposed a hybrid model for the life-cycle assessment of environmental effects in construction products. Junnila et al. (2006) showed a comprehensive list of inputs (energy, materials) and outputs (emissions, waste) in modern American and European building projects. This quantitative assessment is necessary to make at least a broad estimation of environmental costs. Kanda et al. (2007) discussed a method to incorporate carbon dioxide emissions in cost-benefit analyses.

After all the costs of failure have been determined, the total expected cost is calculated by substituting the initial cost, the cost of failure, and the estimated probability of failure in Equation 3-3. This result typically is a poor estimate of the expected total cost, as discussed in Chapters 1 and 2, because the true probability of failure is considerably higher than the nominal value. Surprisingly, very few authors mention this issue. The available solutions presented in the following section consider only

nominal reliabilities, but Chapter 4 contains some alternatives to estimate true probabilities of failure.

3.4 Single Component / Single Failure Mode

The optimization problem called URBEO, defined in Equation 3-3, is valid only for systems with a single component and a single failure mode. In general, civil infrastructure projects have multiple components, and each of them has multiple failure modes. The reliability of a structure with multiple components is called system reliability. Many studies have addressed the problem of combining component reliabilities, but this is one of the most difficult aspects of RBDO. Still, a single-component simplification can be useful to illustrate the process or design simple systems.

Basically, there are two types of solutions for the optimization problem. As mentioned previously, the reliability calculation is an optimization problem itself. But the reliability result is required in each iteration of the design optimization process. So there are two nested optimization calculations that typically require relatively large computational efforts. For example, if the reliability calculations are done with direct MCS and the design optimization uses heuristics, the problem is practically impossible to solve with the available technology.

Other combinations of methods can be used to reduce the number of calculations, but there are some ways to avoid the double loop problem. One way is to consider a single resistance and a single load variable with normal or lognormal distributions, so that the reliability index can be obtained directly from Equations 2-6 and 2-7.

Unfortunately, it is necessary to establish a relationship between initial cost and safety, which is not straightforward.

In some studies (Turkstra 1967, Rosenblueth 1986, Kanda and Ellingwood 1991), the initial cost has been determined as a linear function of the global safety factor, as given below, but this is not very accurate for common geotechnical structures:

$$C_I = C_0 \left(1 + k \left(\frac{Q_d}{F} - 1 \right) \right) \quad (3-5)$$

in which C_0 = initial cost when $Q_d = F$, Q_d = design resistance assumed to be equal to the design load effect, F = reference load effect, and k = normalized initial cost ratio.

3.4.1 Closed-Form Solutions

Rosenblueth (1976) proposed a method to obtain the optimum probability of failure when the capacity and demand are normally distributed and the initial cost increases linearly with the capacity. It involves solving iteratively Equation 3-6 to find the optimum reliability index and the optimum safety factor.

$$\frac{g}{k} \frac{FS \cdot COV_Q^2 + COV_F^2}{(FS^2 \cdot COV_Q^2 + COV_F^2)} \phi(\beta) = 1 \quad (3-6)$$

in which g = normalized failure cost = C_F/C_0 , FS = factor of safety, COV = coefficient of variation, and $\phi(\cdot)$ = standard normal probability density function.

This approach is not a closed-form solution, but it is the predecessor of other solutions and it only requires one iteration loop.

Kanda and Ellingwood (1991) derived closed-form solutions for the optimum reliability when the load and resistance probability distributions are assumed to be normal, lognormal, or Gumbel (Figure 3-1).

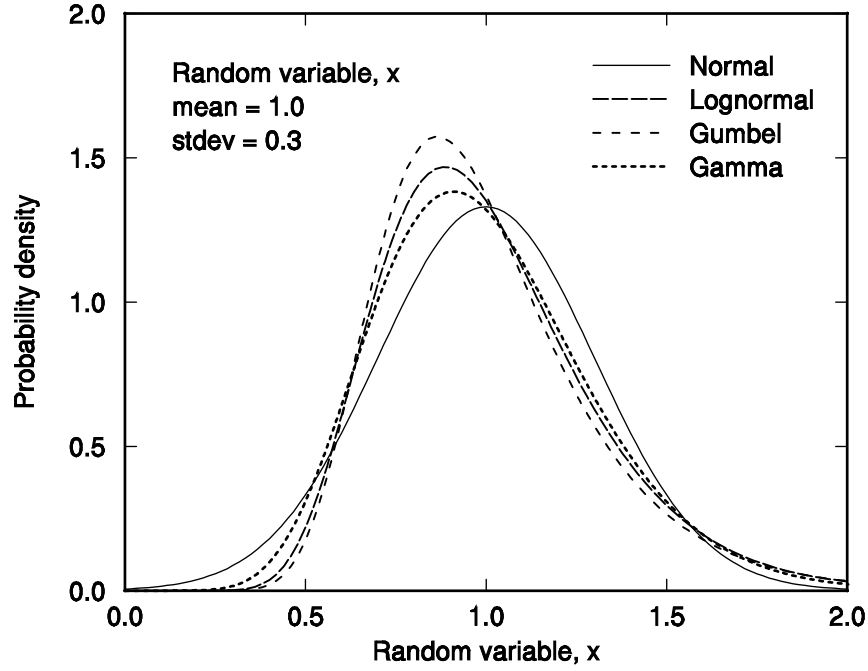


Figure 3-1. Common distributions in reliability-based design

The closed-form solution for a single load with Gumbel distribution and deterministic resistance is given below:

$$\beta_{\text{opt}} = -\Phi^{-1}(1 - \exp\{-\exp[-a(s-b)]\})$$

$$s = m_F + \ln\left(\frac{g}{k \text{ COV}_F} - 0.33\right) \frac{s_F}{1.28} \quad (3-7)$$

in which s_F = standard deviation of the load, m_F = mean load, $a = 1.28 / s_F$, and $b = m_F - 0.45 s_F$.

When the capacity and demand terms have lognormal distributions, the optimum reliability index is:

$$\beta_{\text{opt}} = -(\ln \zeta)^{0.5} + \left(\ln \zeta + 2 \ln \left[\frac{g(1 + \text{COV}_F^2)^{0.5}}{k \left[2 \pi \ln(\zeta) (1 + \text{COV}_Q^2) \right]^{0.5}} \right] \right)^{0.5} \quad (3-8)$$

in which $\zeta = (1 + \text{COV}_Q^2)(1 + \text{COV}_F^2)$.

The closed-form solution for normal distributions (Eq. 3-9) requires the linearization shown in Appendix A.

$$\beta_{\text{opt}} = \left(2 \ln \left(\frac{g}{k (2 \pi)^{0.5} \alpha \text{COV}_F} \right) \right)^{0.5} \quad (3-9)$$

in which α = separation coefficient.

This simplification introduces the separation coefficient, α , which can have values between 0.707 and 1.0, depending on the values of the load and the resistance COVs.

In most cases, assuming that $\alpha = 0.75$ does not introduce a significant error in the solution. However, the separation coefficient can be computed exactly using Equation A-1 or tables.

Note that the three closed-form solutions have the ratio of g/k in their equations. It is reasonable to expect that the optimum reliability index increases with the cost of failure and decreases with the cost of additional safety. Moreover, the coefficient of variation of the load, COV_F , multiplies the parameter k in the equations for Gumbel and normal distributions. This condition indicates that, as the load uncertainty

increases, the reliability requirements should be lower, all else being constant. The relationship between load uncertainty and optimum safety is not obvious. One interpretation suggests that investing in safety becomes a less efficient strategy to achieve minimum life-cycle costs when the behavior of a structure is highly uncertain. The same idea applies to the variability of the resistance according to the results shown in Chapter 5.

3.4.2 Gradient Based Algorithms

Another approach to avoid the double optimization loop in URBEQ is the use of the response surface method (RSM) to approximate the reliability of a particular design. The main objective of the RSM is to offer an inexpensive approximation (e.g. algebraic equation) of an implicit performance function (e.g. results from finite element models) when a very large number of calculations are required. The basic idea is to perform the expensive analysis for a small number of points in the design space. Then, use a polynomial interpolation to fit the results with a hyper-surface (Figure 3-2). Finally, use the fitted equation to estimate the results in other points of the design space.

One application of RSM is to calculate the reliability of a component when the performance function is implicit. The reliability of a structure is calculated using direct MCS in conjunction with the RSM approximation instead of the expensive implicit function. Wong (1985) showed an example of this approach for slope stability analysis using finite elements.

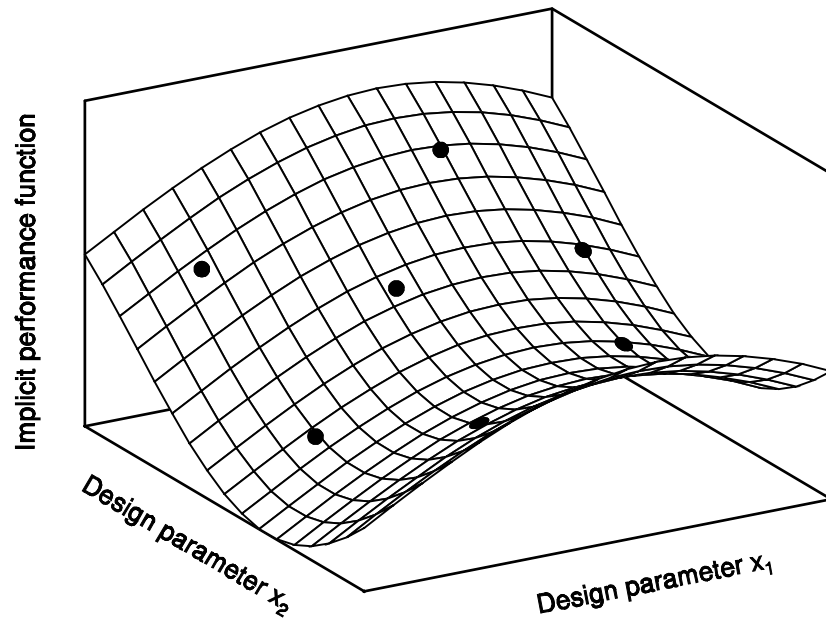


Figure 3-2. Illustration of the response surface method

Xu and Low (2006) solved a similar problem using FORM. They used the RSM to obtain an inexpensive approximation of slope stability results from finite elements. Then, the equation of the fitted hyper-surface was used as an explicit performance function to compute the reliability of an embankment according to FORM.

Similarly, RSM can be used to simplify calculations in RBDO. The process for this application is identical to the previous case, except that the approximation is used for the reliability instead of the implicit performance function. The first step is to calculate the reliability of several points in the design space using conventional methods (MCS, FORM, etc.). Then, a response surface is fitted to those points. This explicit function is used to estimate the reliability at any point of the design space, eliminating one of the loops of RBDO. Finally, a nonlinear optimization engine can minimize the objective function in a single loop. One advantage of this method over the closed-form

solutions is that the original design variables are used. There is no need to simplify the problem to one load and one resistance random variables. Gasser and Schuëller (1997) showed an example of this approach for the optimization of a fixed offshore platform.

3.5 Multiple Failure Modes

The problem of multiple components or multiple failure modes is not an easy one. The reliability of a system is defined as the probability that no failures occur during a period of time. In a time-invariant format, the system reliability is given as below:

$$r_{\text{sys}} = P((g_1 > 0) \cup (g_2 > 0) \cup \dots (g_n > 0)) \quad (3-10)$$

in which g_i = performance function of i^{th} component and n = number of components.

In principle, it is possible to estimate the reliability of a system with multiple components for different configurations. Grigoriu and Turkstra (1979) presented closed-form equations to obtain the reliability of systems with any number of correlated, identical components in series or parallel arrays. When the reliabilities of the components are not identical, the system reliability for independent components is defined below:

$$r_{\text{sys}} = 1 - \prod_{i=1}^n p_{fi} \quad \text{for parallel systems} \quad (3-11)$$

$$r_{\text{sys}} = \prod_{i=1}^n (1 - p_{fi}) \quad \text{for series systems}$$

in which n = number of components and p_{fi} = probability of failure of i^{th} component.

Real structures have more complex combinations of components and failure modes. Typical systems have components with different reliabilities, and each component can have several correlated failure modes.

If we consider a simple case with only two failure modes, each failure mode will have a failure surface in the design space (Figure 3-3).

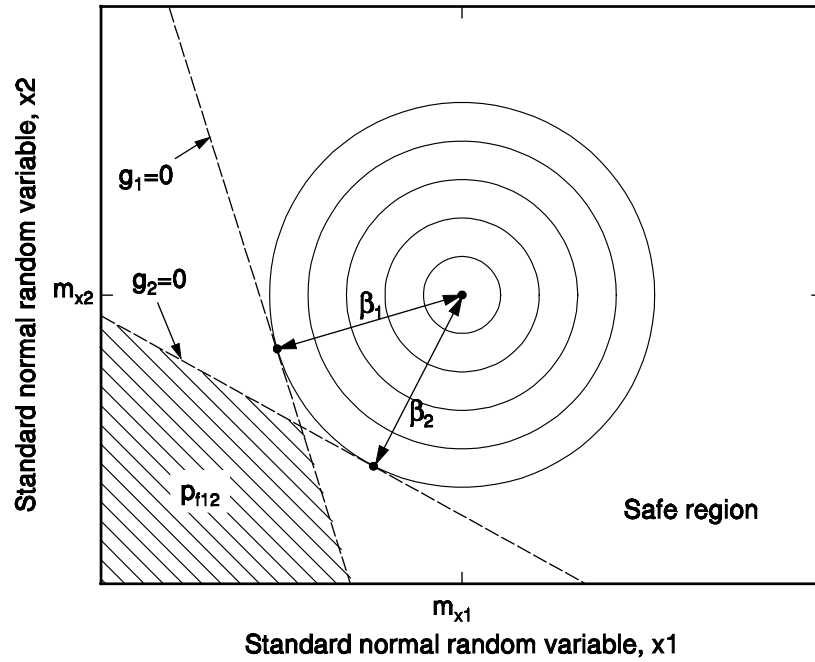


Figure 3-3. System reliability for two failure modes or components

Then the reliability of the system is equal to the integral of the multivariate probability density function over the safe domain, as given below:

$$r_{\text{sys}} = \int_{(g_1 < 0) \cap (g_2 < 0)} f(x) dx \quad (3-12)$$

in which g_i = performance function for i^{th} failure mode, $f(\cdot)$ = joint probability density function, and x = vector of all random variables.

There are no closed-form solutions for this apparently simple problem, even if the failure surfaces are planar (Phoon 2008). The best approximations are obtained with numerical simulations but they are computationally expensive and time-consuming. Often times, it is useful to compute bounds for the system reliability. A number of studies describe methods to compute bounds for system reliability using different techniques (Ditlevsen 1979, Song and Kiureghian 2003, Phoon 2008).

Mendell and Elston (1974) presented a point estimate of the intersection of two failure modes (hatched area in Figure 3-3). The probability that both failure modes occur is given by Eq. 3-13:

$$p_{f12} \approx \Phi \left[\frac{\rho_{12} a_1 - \beta_2}{\sqrt{1 - \rho_{12}^2} a_1 (a_1 - \beta_1)} \right] \Phi(-\beta_1) \quad (3-13)$$

$$a_1 = \frac{1}{\Phi(-\beta_1) \sqrt{2\pi}} \exp\left(-\frac{\beta_1^2}{2}\right)$$

in which ρ_{12} = correlation coefficient between failure modes.

Therefore, an approximate of probability of failure for the system can be calculated using the equation below:

$$p_{sys} \approx p_{f1} + p_{f2} - p_{f12} \quad (3-14)$$

Intuitively, the correlation coefficient between failure modes must be positive in most cases, because they depend on the same set of loads.

However, in life-cycle cost optimization, every failure mode and every component failure can have different consequences. Therefore, the expected cost of failure must regard each failure separately. The complexity of the problem increases even more

because some failures depend on the behavior of other components (e.g. redundant elements). In some cases, the expected cost of failure requires the use of conditional probabilities for more realistic estimates. When only a few failure modes are dominant, it is possible to implement an approach that addresses the correlation problem. The first step is to define all possible failure states, which are all possible combinations of component failures and failure modes. Then, estimate the probability to reach each state according to the probability of failure of each component and their correlation structure. The expected cost of failure is obtained by adding all the products of the cost of each state and its probability as given below:

$$E(C_F) = \sum_{i=1}^n C_i p_{si} \quad (3-15)$$

in which C_i = failure cost of i^{th} state, p_{si} = probability of reaching state i , and n = number of possible failure states.

This method is not practical when many failure modes exist as the number of states increases rapidly. Also, the probability calculation of each state is not straightforward because some states may exclude the possibility of further failures, depending on the time of occurrence. However, the approach is a reasonable approximation for problems that do not include time-dependent variables.

3.6 Summary

Many methods for structural optimization with different objectives exist within RBDO. Problems with a single life-cycle cost objective are consistent with decision theory and cost-benefit analysis principles. In this approach, there are no reliability

constraints, and upper and lower bounds for design variables are optional to prevent unreasonable values. It is proposed herein to call this type of problem an unconstrained reliability-based economic optimization (URBEO) to distinguish it from other problems. A number of authors have recognized this alternative as the best rational approach to calculate target reliability levels, although it is still too complex for practical applications. The available approaches were described, including comments on their assumptions and limitations. Probably the most difficult challenge is the combination of several components with different failure modes and time-variant formulations.

Another critical issue is the use of nominal probabilities of failure to compute the expected cost of failure. So far, the available optimization models do not attempt to estimate true probabilities of failure, leading to significant bias in the results.

CHAPTER 4

TRUE VS. NOMINAL RELIABILITY

4.1 Introduction

It is well known in the structural design community that the nominal probability of failure obtained from a reliability analysis can be drastically different from the true probability of failure. Actual failure rates can be orders of magnitude larger than the nominal values (Brown and Yin 1988). Even so, the literature on target reliability levels rarely discusses this issue. Moses (2001) noted that this discrepancy is one of the reasons why LRFD codes avoid referring to the probability of failure and use reliability indices instead.

The wide gap between nominal and true values is caused by errors in the design, construction, or use of a structure, which are sometimes called “human errors”. Results from probabilistic methods are based on the assumption that the design, construction, and use of structures are flawless, models are perfect, and all possible failure modes have been considered. Usually, the human and organizational factors are called “soft” or “extrinsic”, while the natural variability of parameters and knowledge uncertainties are called “hard” or “intrinsic”.

Some engineers believe that extrinsic factors are outside the scope of engineering. Blockley (1999) asserted that “... dealing with the total system uncertainty, which includes the chance of human error, is daunting because the ability of humans to do the unexpected is almost infinite.”

Typically researchers in other fields, such as psychology and social sciences, study this subject. However, engineers must estimate, at least approximately, the influence of “human errors” to obtain more significant results and understand the implications of all the uncertain factors during the life of structures. In current practice, the use of nominal values impedes taking full advantage of probability theory. Certainly, probabilities in geotechnical engineering must be regarded as degrees of belief (Bayesian approach) rather than frequency of events (frequentist approach), because every project is unique. Still, in the long run, the number of observed failures should be consistent with the computed probabilities, which is not the case currently. The need to evaluate extrinsic factors is clear, but the best way to do it is not clear. Before presenting a new method to estimate true probabilities of failure, the available approaches regarding human errors will be discussed.

4.2 Definitions

A simple Venn diagram (Brown et al. 2008) can help understand the implications of considering human errors in the design process. Figure 4-1 (a) shows the failure set, F , and the safe set, S , with boundary G^* . The asterisk in the designation of the boundary means that the true limit is unknown. The second diagram divides the design space, D , into two new sets: (a) structures with errors, E , and (b) structures without errors, R , with boundary P . It should be clear that the union of F and S is equal to D , and the union of E and R is also equal to D .

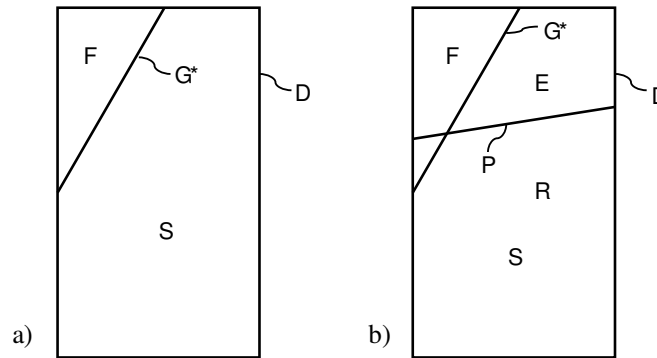


Figure 4-1. Venn diagrams showing: (a) safe and failure sets with boundary G^* , and (b) error set, E, and error-free set, R, with boundary P (Brown et al. 2008)

Brown et al. (2008) suggest that most structures have errors but do not fail. Other authors point out that engineered systems have measures to prevent failures when a single error occurs. Catastrophic failures typically require a sequence of human errors that exceed the safety measures. The literature contains numerous examples of disasters caused by multiple errors such as Chernobyl, Three Mile Island, the offshore platform Piper Alpha, the space shuttle Challenger, etc. In other cases, it may be more difficult to distinguish a failure caused by error from a failure caused by natural variability. A relatively small design error may not lead to failure, unless an extreme loading event occurs.

4.3 Types of Errors and Failures

The word error comes from the Latin word *errare*, which means “to wander or go astray”. Some common definitions of error found in dictionaries and encyclopedias include:

- A wandering or deviation from the right course or standard.

- An act that unintentionally deviates from what is correct, right, or true.
- An act that through ignorance, deficiency, or accident departs from or fails to achieve what should be done.
- In mathematics: the difference between a computed or measured value and a true or theoretically correct value.
- In engineering: a difference between the desired and actual performances or behaviors of a system or object.

The previous definitions imply that a solution containing errors can be at any distance from the “correct value”. Then, it is necessary to make a distinction between small errors and significant ones. Significant deviations are also called “gross errors”.

Ellingwood (1987) states that a precise definition of “gross error” can not exist, because there is uncertainty in the bounds of what is considered “acceptable practice”. He also points out that some errors may actually contribute to structural safety. These two ideas are essential to the approach proposed herein.

The causes and types of errors have been discussed extensively. Ellingwood (1987) describes three categories:

- Errors of concept (stupidity, ignorance)
- Errors of execution (carelessness, forgetfulness, negligence)
- Errors of intention (venality, irresponsibility)

The last category would be inappropriate if we consider that errors are unintentional.

Brown et al. (2008) define three categories of failures caused by errors:

- Technical nature (TU). These failures are related to the technical aspects of the design and construction, and they generally can be reduced with additional professional resources. The Hyatt Regency Hotel walkway failure and the Minneapolis bridge collapse are good examples.
- Technical ignorance (UT). On a personal level, they result from lack of engineering knowledge. On a community level, they arise from previously unknown failure modes (e.g. the Tacoma Narrows bridge collapse).
- Non-technical errors (U). These are errors caused by organizational or other non-technical aspects.

Researchers can also classify errors and failures according to the time of occurrence, consequences of failure, detection probability, etc. These classifications are useful to identify hazards and to develop preventive measures. The effect of preventive measures such as quality assurance programs, personnel training, supervision levels, etc. can be evaluated quantitatively if enough statistical information is available.

It is important to note that many errors are detected after a failure occurs. Systems that perform adequately are rarely reviewed after construction. It seems reasonable to believe that most errors on the safe side, and small errors that do not lead to failure, will go undetected unless they affect the budget significantly.

4.4 Modeling Human Errors

Most studies conclude that human errors are responsible for 80 to 90% of failures in buildings (Brown and Yin 1988). The failure rate or annual probability of failure is about 10^{-2} to 10^{-3} for foundations and 10^{-4} to 10^{-5} for dams (Baecher and Christian 2003). These figures do not specify the proportion of ultimate and serviceability

failures. However, Brown and Yin (1988) report data suggesting that approximately 60% of the failures in bridges and buildings are ultimate and 40% are functional.

4.4.1 Quality Management Assessment System

Probabilistic risk assessment (PRA) is a common methodology to evaluate risks associated with complex engineering systems. However, it is necessary quantify the risk associated with human activities using probability theory.

Bea (2006) described a method to assess quantitatively the influence of extrinsic factors in the failure rate of geotechnical designs. The quality management assessment system (QMAS) evaluates seven components of the human and organizational factor, including interfaces, environments, structure, equipment, procedures, organizations, and operators. Each component is evaluated and graded on a scale from one to seven, as shown in Figure 4-2.

Each grade corresponds to a performance shaping factor (PSF). The global PSF is equal to the product of the seven component PSFs. Finally, the average failure rate related to extrinsic factors is multiplied by the global PSF to obtain a specific extrinsic failure rate for a particular process or system. According to this method, each component of the QMAS can increase or decrease the expected failure rate by three orders of magnitude. The method was originally developed for offshore structures (Bea 2000), but it can be adapted to evaluate the extrinsic factors in the design of geotechnical structures.

QMAS Grade		PSF for PRA	
Outstanding exceeds all standards and requirements	1	10^{-3}	Extreme positive influence
Excellent	2	10^{-2}	Moderate positive influence
Very good	3	10^{-1}	Minor positive influence
Good, average exceeds all standards and requirements	4	10^0	Negligible influence
Below average	5	10^1	Minor negative influence
Poor	6	10^2	Moderate negative influence
Very poor, does not meet any standard or requirement	7	10^3	Extreme negative influence

Figure 4-2. Quality management assessment system (QMAS) grading (Bea 2006)

4.4.2 Errors as Random Variables

Probabilistic risk assessment (PRA) and other similar methods take into account risks associated with human errors, but they can not be applied directly to engineering design. Those methods were developed for industrial processes in which human operations are clearly identified, not for general design and construction processes. In structural and geotechnical engineering, every project is different, and the number of possible tasks is almost infinite. Even when the same design problem is given to several engineers, they may use different criteria or models that lead to different solutions.

Other attempts to model human error in design include surveying failure rates in design tasks (Stewart 1992, Melchers 1989). Unfortunately, the available statistical data for design and construction of typical structures is not large enough to offer accurate results. Another frequently proposed model (Bea 2000) involves calculating the probability of failure using Bayes' theorem as:

$$P(F) = P(F_e | E) P(E) + P(F_n | E) P(E) + P(F_n | \bar{E}) P(\bar{E}) \quad (4-1)$$

in which $P(i | j)$ = conditional probability of i given j , F_e = failure caused by an error, F_n = failure caused by natural variability, E = error occurs, and \bar{E} = error does not occur.

The problem with this approach is that, as mentioned previously, the definition of error is arbitrary. Each type of error can have a very different effect on the performance function. The engineer would have to select a representative error and then estimate the probabilities in the equation.

A better way to deal with this problem is to evaluate the effect of errors on the performance function. Errors that increase or decrease the reliability of the system will be called safe and unsafe, respectively. Assuming that errors are unintentional, there is no reason to believe that unsafe errors are more frequent than safe errors or vice versa. In reality, each error will alter the mean of the performance function, but we can not know *a priori* (before detecting the error) its magnitude or sign. Also, when all deviations from the "correct value", small and large, are taken as errors, the effect can be regarded as a higher variability (Figure 4-3). The magnitude of an error is an unknown that can be treated as a random variable. This approach will be called human error as a random variable (HERV).

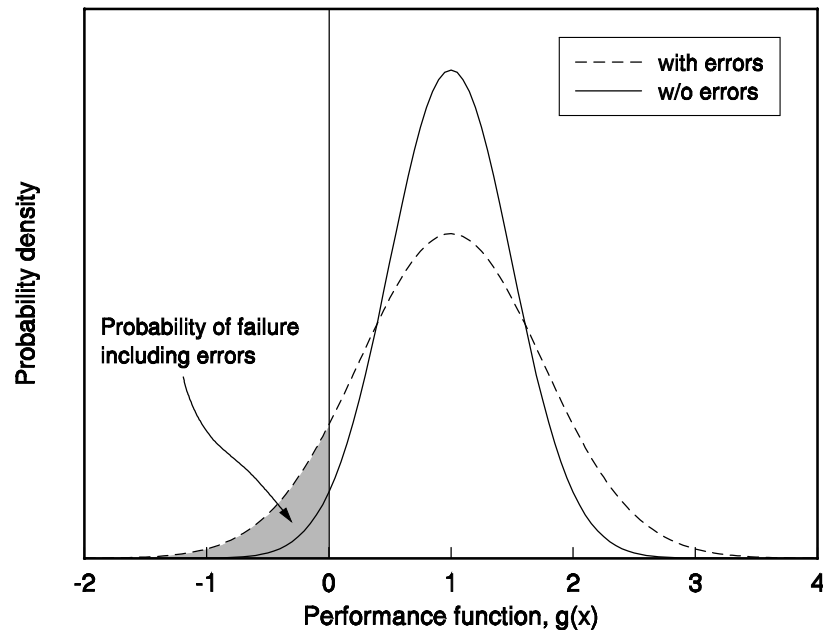


Figure 4-3. Uncertainty of performance function with and without errors

Vick (2002) used the same approach to estimate the probability of failure including model errors. Vick separates the variability of the performance caused by models into model error and model uncertainty. Model error is the variability associated with the simplifications assumed in a particular model. On the other hand, model uncertainty accounts for the different predictions that result from using different models. He argues that the concept of model uncertainty is necessary, because there will always exist several models to predict a single phenomenon. However, different models may use different parameters or equivalent physical parameters with different statistical properties. In the author's opinion, there is no such thing as a correct model. All models are simplifications that carry associated biases and errors. It is possible to estimate errors for each model by comparing observed and predicted behaviors. Then, based on statistical measures, we should be able to assess that a model is more accurate than another when predictions are closer to observed values.

Considering errors as random variables is not an unreasonable approach after observing the results of prediction surveys, which are exercises where a group of professionals are asked to predict the behavior of a structure (load at failure, deformations for a given load, etc.). Then, the predictions are compared with experimental results.

In one prediction survey (Hynes and Vanmarke 1976), seven internationally-known geotechnical engineers estimated the failure height of an embankment. When the embankment was built, the true failure height was determined. Most probably, none of the calculations contained “gross errors”. However, the results were highly variable.

If errors are normally distributed and independent from the intrinsic variability, the true mean and variance of the performance function, g , are given by Equations 4-2 and 4-3.

$$m_t = m_g + m_e \quad (4-2)$$

$$s_t^2 = s_g^2 + s_e^2 \quad (4-3)$$

in which m_g = nominal mean of g , m_e = mean error, s_g^2 = nominal variance of g , and s_e^2 = variance of error.

As mentioned previously, the mean value of the error should be equal to zero. Then, the nominal and true reliability indices are given by:

$$\beta_n = \frac{m_g}{s_g} \quad (4-4)$$

$$\beta_t = \frac{m_g}{(s_g^2 + s_e^2)^{0.5}} \quad (4-5)$$

Combining Equations 4-4 and 4-5 results in:

$$\frac{s_e}{m_g} = \left(\frac{1}{\beta_t^2} - \frac{1}{\beta_n^2} \right)^{0.5} \quad (4-6)$$

The ratio of the standard deviation to the mean of the performance function is a useful quantity, because it has no units and can be regarded as a coefficient of variation because of errors. Equation 4-6 can be written also as:

$$\text{COV}_e = \frac{s_e}{m_g} = \frac{1}{\beta_t} \left(1 - \frac{1}{(\beta_n / \beta_t)^2} \right)^{0.5} \quad (4-7)$$

In Equation 4-7, the ratio of the nominal to the true reliability indices can be used to obtain the variability of errors for a certain type of design problems. For example, if a structural system or component with a single failure mode is designed for a target reliability index of 3.5, while the observed, true reliability index is 2.0, the coefficient of variation because of errors would be 0.41. Figure 4-4 shows this relationship for different values of the nominal reliability index ratio. Figure 4-5 shows the same relationship in a plot of true vs. nominal reliability indices and varying coefficient of variation of errors.

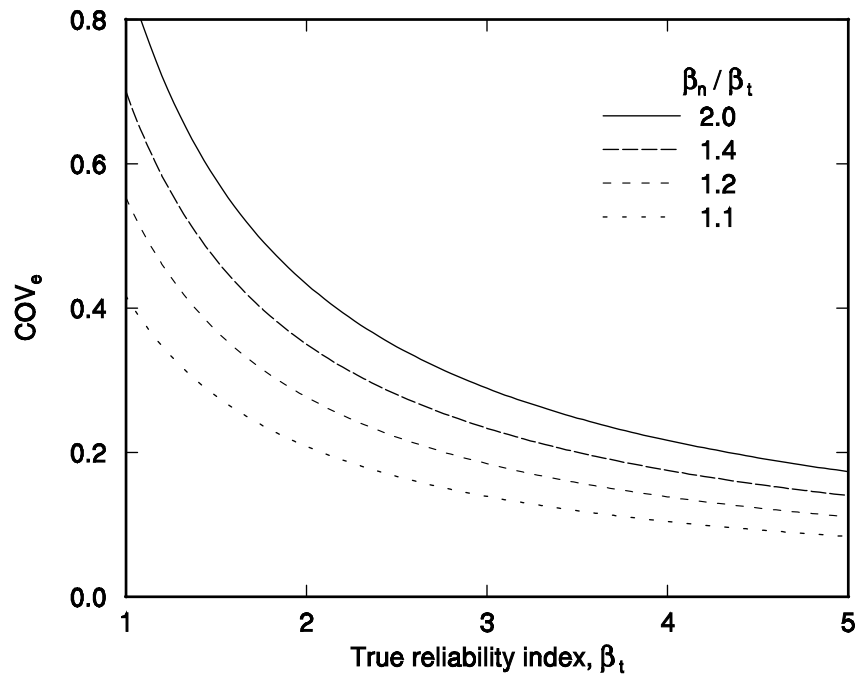


Figure 4-4. Coefficient of variation of errors (COV_e) as a function of the true and nominal reliability indices

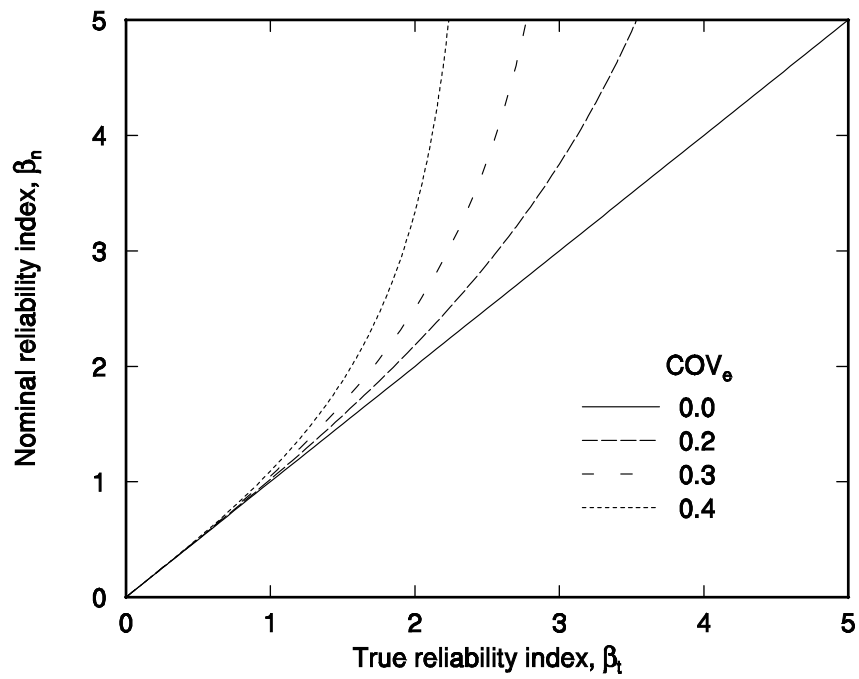


Figure 4-5. True vs. nominal reliability index for varying COV_e

If the component is designed using a mean factor of safety, FS, the performance function can be taken as (FS – 1).

When the nominal and error variability of the performance function are correlated, the relationship is given by:

$$\frac{s_e}{m_g} = -\frac{\rho}{2\beta_n} + \sqrt{\frac{1}{\beta_t^2} - \frac{1}{\beta_n^2} + \left(\frac{\rho}{2\beta_n}\right)^2} \quad (4-8)$$

in which ρ = correlation coefficient.

Other equations can be derived using the concept of the coefficient of variation because of errors, COV_e . Adding an error term to Equation 2-6 results in:

$$\beta_t = \frac{FS - 1}{\left(FS^2 COV_Q^2 + COV_F^2 + (FS - 1)^2 COV_e^2 \right)^{0.5}} \quad (4-9)$$

Equation 4-9 is an approximation of the true reliability index assuming that the capacity, load, and error terms are normally distributed, independent, random variables. The relationship between the coefficient of variation of the error and the reliability index is shown in Figure 4-6.

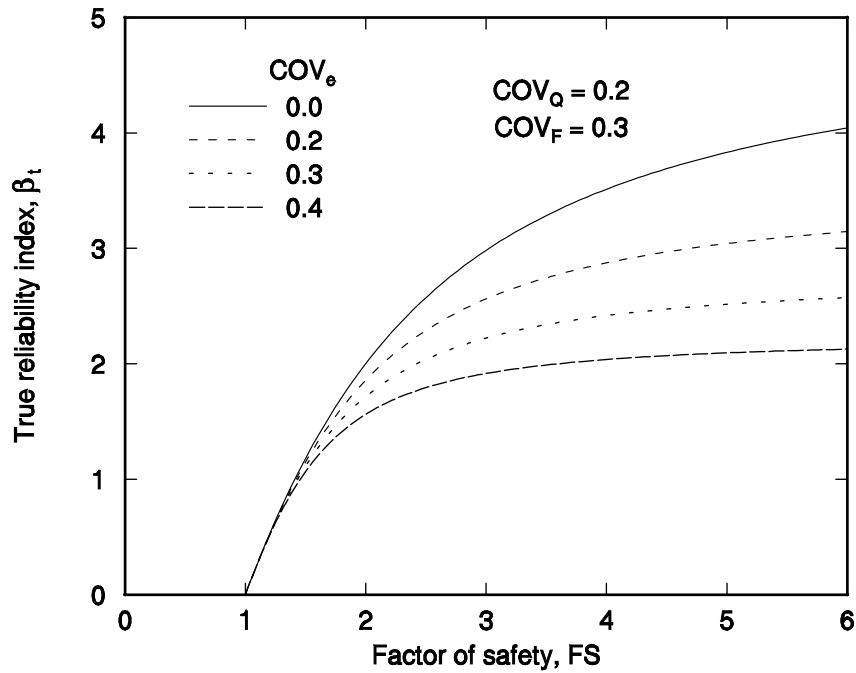


Figure 4-6. True reliability index for normal Q and F

If there are physical limits for the value of the performance function, other distributions, such as the lognormal, can be more accurate than the normal distribution. There are no data available indicating that errors have a particular distribution or bias, and therefore the normal distribution with zero mean should be used. In contrast to errors, load and resistance terms are typically extreme values or products of independent random variables. In many cases, the solution assuming lognormal distributions of the load and resistance (Eq. 2-7) is more accurate for real projects. Using Equation 2-7 to calculate the nominal reliability index in Equation 4-6, and solving for the true reliability index, gives the results shown in Figure 4-7.

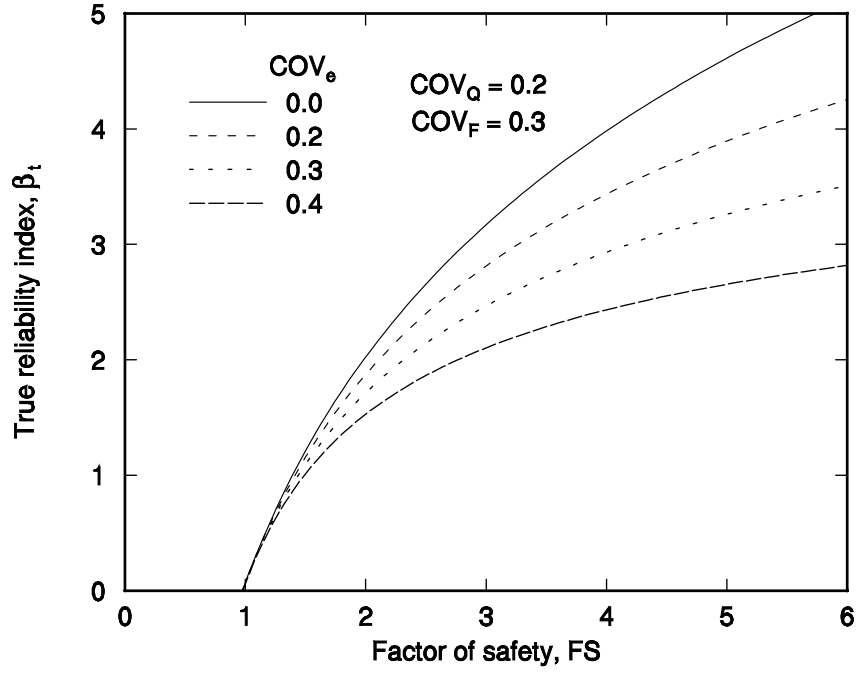


Figure 4-7. True reliability index for lognormal Q and F.

Other approximations that include human errors can be derived following the simplified procedure used for closed-form solutions. If the initial cost is taken as a linear function of the safety factor, and the probability of failure is calculated with Equation 4-9, the expected cost is:

$$E(C_T) = C_0 (1 + k (FS - 1)) + g C_0 \Phi(-\beta_t) \quad (4-10)$$

in which C_0 = initial cost when $FS = 1$, k = linear cost coefficient, and g = normalized cost of failure = C_F/C_0 .

Taking the derivative of Equation 4-10 with respect to FS , and setting it equal to zero, results in the following:

$$\frac{g}{k} \frac{FS \cdot COV_Q^2 + COV_F^2}{(FS^2 \cdot COV_Q^2 + COV_F^2 + (FS - 1)^2 COV_e^2)} \phi(\beta_t) = 1 \quad (4-11)$$

in which $\phi(\cdot)$ = standard normal probability density function

Similarly to the solution without human errors, Equations 4-11 and 4-9 must be solved iteratively to determine the optimum safety factor and optimum true reliability index.

4.4.3 Error Ratio

Another closed-form solution requires a new parameter called error ratio, v . The error ratio, which is regarded as constant for a specific type of project, is simply the ratio of the nominal and true reliability indices, as given below:

$$v = \frac{\beta_n}{\beta_t} \quad (4-12)$$

The accuracy of this model is limited, because it implies that the magnitude of the error is proportional to the nominal variability of the performance function. For comparison purposes, assuming that the load and the resistance have lognormal distributions, then the true reliability index is given by:

$$\beta_t = \frac{\ln \left(m_Q / m_F \sqrt{(1 + \text{COV}_F^2) / (1 + \text{COV}_Q^2)} \right)}{v \sqrt{\ln \left[(1 + \text{COV}_F^2)(1 + \text{COV}_Q^2) \right]}} \quad (4-13)$$

Equation 4-13 is plotted for different values of v in Figure 4-8. For higher values of the v ratio, the true reliability index decreases as expected.

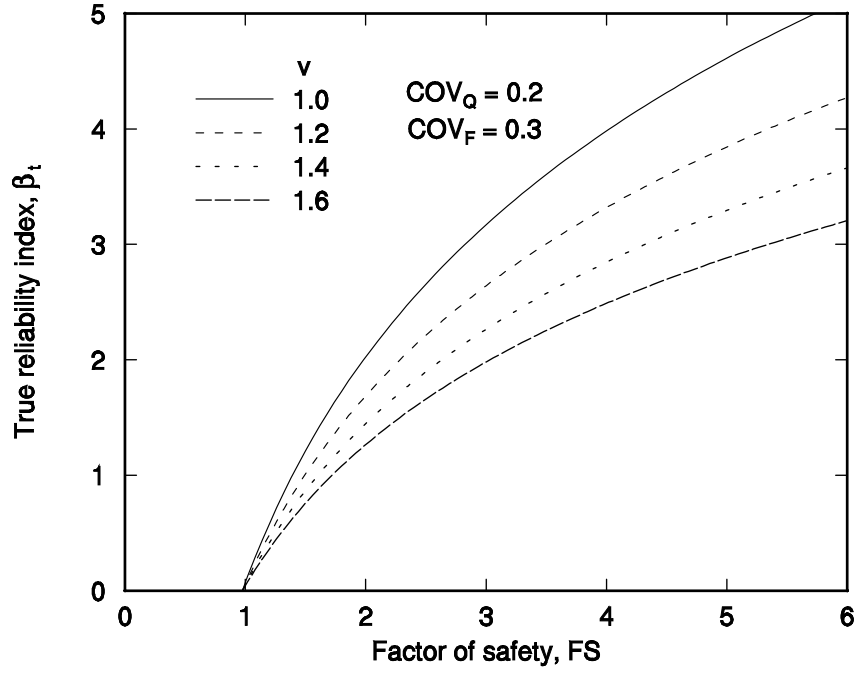


Figure 4-8. True reliability index for lognormal Q and F

Substituting Equation 4-13 in Equation 4-10, taking the derivative with respect to FS, and solving for the optimum nominal reliability index, results in the following:

$$\beta_{n_opt} = -v^2 \sqrt{\ln(a b)} + v^2 \sqrt{\ln(a b) + \frac{2}{v^2} \ln \left[\frac{g \sqrt{b/a}}{k v \sqrt{2\pi \ln(a b)}} \right]} \quad (4-14)$$

in which $a = (1 + \text{COV}_Q^2)$ and $b = (1 + \text{COV}_F^2)$.

Note that Equation 4-14 reduces to Equation 3-8 when the ratio v is equal to one. The change of β_{n_opt} for different values of v and g is shown in Figure 4-9. Another useful plot of the same equation is shown in Figure 4-10.

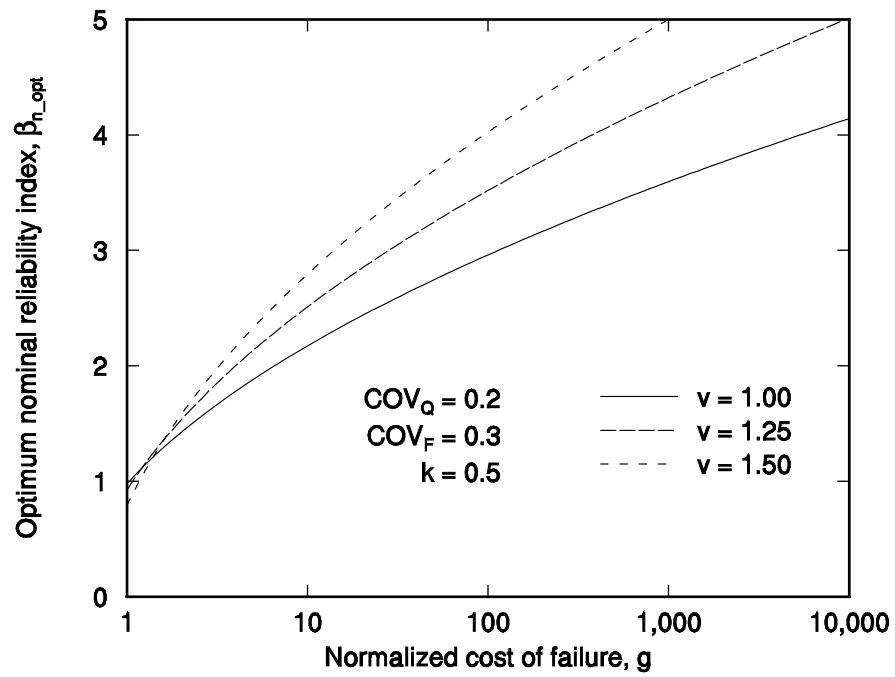


Figure 4-9. Optimum nominal reliability index vs. failure cost for lognormal Q and F

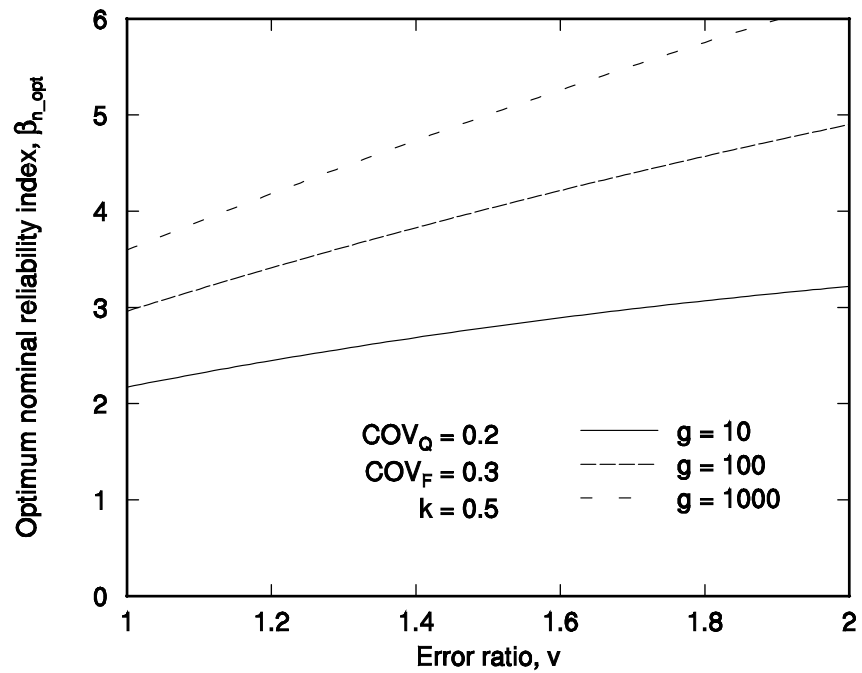


Figure 4-10. Optimum nominal reliability index vs. error ratio for lognormal Q and F

Note that the optimum nominal reliability index increases as the normalized cost increases. This behavior is intuitive, because the probability of failure should be lower for structures with high cost of failure (e.g. important buildings). Obviously, the optimum nominal reliability is higher when the error ratio, v , increases.

Equation 4-14 is an important result, because it can include a simplified form of the costs, as well as human errors, to estimate optimum reliability indices. However, it has the same limitations as the closed-form solutions presented in Chapter 3, because it is necessary to estimate the error ratio, v , in addition to the cost constants, g and k . Moreover, one has to assume lognormal distributions for Q and F and estimate their COVs.

4.5 Design Strategies

The two approaches considered herein for human error modeling, QMAS and HERV, can be used directly to estimate true reliability indices. Both approaches must be calibrated carefully with observed failure rates for each type of project. Because of the lack of statistical data, some simplifications are pertinent, such as the assumption of normal distribution of errors in HERV.

These results can be applied, not only in design optimization, but also in the traditional trial and error approach. For example, if only an approximate result is needed, Equation 4-9 can be used directly, assuming that the variability of the load, resistance, and error terms are known. Target reliability indices for this case must be related to the actual failure rates, not to the typical nominal values mentioned in subsection 2.3.2.

The design optimization problem stated in Chapter 3 requires an estimate of the true probability of failure as a function of the design variables. By using HERV in conjunction with an optimization algorithm, it is possible to search for the set of design variables that minimize the life-cycle cost of a structure. In each iteration, the true probability of failure is calculated with Equation 4-5 or 4-9, assuming that the coefficient of variation because of errors is constant (i.e. does not depend on the design variables) for a specific type of structure.

Strategies that regard life-cycle costs are becoming more attractive for structural designers as the cost of resources varies depending on the location and time. For example, several developing countries adopt codes and standards from developed countries, where the consequences of failure are quite different. In some cases, it may be better to accept lower safety levels in favor of increased infrastructure (Sánchez-Silva and Rosowsky 2008). In addition, engineers require a rational framework to compare similar designs that include different construction methods, materials, and local conditions.

Bea (2006) argues that increasing the capacity of a structure may not improve the reliability related to extrinsic factors. This might be true in some, but not all, cases. Consider a foundation design in which human errors lead to an inexplicably high estimated bearing capacity. If a typical safety factor (i.e. 2 to 3) is used, failure is likely to occur, but if the safety factor is higher than usual, it may or may not prevent failure, depending on the magnitude of the error.

Bea (2006) also notes that there are three categories for strategies that improve structural reliability: proactive, reactive, and interactive. Within the first category, achieving robustness is one of the most important strategies. A precise definition of

robustness is not available, but the general notion implies that a robust structure will perform adequately under a wide range of operating conditions. Four factors can affect the robustness of a structure: (a) configuration, (b) ductility, (c) excess capacity, and (d) appropriate correlation. In a life-cycle cost optimization framework, the optimization algorithm only finds the best set of random variables to provide excess capacity. Probabilistic methods can assist engineers to achieve minimum life-cycle cost designs, but they still have to possess enough experience and good judgment to select efficient structural systems, materials, construction methods, quality control methods, etc. Fortunately, different designs can be evaluated rationally by comparing their expected life-cycle cost.

The reactive and interactive approaches also can be applied to geotechnical design. Their goal is to detect, correct, and reduce the consequences of failure. Therefore, monitoring systems, reinforcement methods, maintenance, and repair techniques belong to these categories. The economic impact of such strategies can be included in the life-cycle optimization framework. However, it is difficult to assess their effectiveness, because natural variability and extrinsic factors also affect their performance.

4.6 Summary

In this chapter, two methods for estimating true probabilities of failure were reviewed. Both methods require calibrations with observed failure rates to account for the occurrence of human errors. In the quality management assessment system (QMAS), the standard failure rate is multiplied by a performance shaping factor (PSF) to include the probability of human error. The PSF can be adjusted according to the quality

assurance policies of the design firm. The second method, called human error as a random variable (HERV), considers the effect of human errors in the performance function. A coefficient of variation of the error is calculated based on the average true and nominal probabilities of failure. The COV because of errors is taken as constant (independent of the design variables), but it can be adjusted for particular types of projects.

A simplified design approach that includes human errors was proposed using closed-form solutions for the true reliability index. This approach requires a target true reliability index to accept or reject the design. Two closed-form solutions were presented: the first assumes normally distributed errors with zero mean, while the second uses a constant ratio of nominal to true reliability index. Both approaches can be used in the cost optimization problem called URBEQ, where true probabilities of failure are needed.

CHAPTER 5

LIFE-CYCLE COST OPTIMIZATION FRAMEWORK

5.1 Introduction

In this chapter, a design optimization framework is presented to solve the problem called unconstrained reliability-based economic optimization (URBEO), as defined in Section 3.2. The main goal of the framework, which is to minimize an objective function, can be achieved using different reliability methods. The methods presented herein are Monte Carlo simulation (MCS) and first order reliability method (FORM) using a spreadsheet. These methods are relatively simple and accurate enough for the task. In general, foundation reliability results obtained with first order second moment (FOSM) or point estimate (PE) methods can have significant errors, because foundation design equations are nonlinear functions of several parameters.

Previous approaches to obtain optimum safety levels were discussed in Chapter 3. Those methods present two significant drawbacks. First, closed-form solutions can not optimize the design variables, because the equations are a function of lumped load and resistance variables. The second and more serious problem is that they use nominal, not real, probabilities of failure.

Two examples also are shown in this chapter. A spread footing on sand and a drilled shaft in cohesive soil are designed using the proposed approach. In both cases, the results for nominal and true probabilities of failure are presented. In the first example, the results for the nominal probability of failure are compared with closed-form solutions. Also, a sensitivity analysis shows the most influential parameters in the

design. Finally, some general comments about the results and convenience of the methods are given.

5.2 Objective function

This framework, like many previous studies, considers a single objective function. Although it is possible to use a multi-objective approach, decision theory does not require multiple objectives. As discussed in Chapter 3, calculation of benefits is rather complex, and therefore minimization of cost functions is a reasonable simplification. Naturally, when the objective function represents costs, rather than net benefits or utilities, it must be minimized, but the procedure is identical otherwise. Examples presented in this section consider a life-cycle cost function (See Chapter 3), which is shown again below:

$$C_T(x) = C_I(x) + C_F(x) p_f(x) \quad (5-1)$$

in which C_T = total cost, C_I = initial cost, C_F = cost of failure, and p_f = probability of failure.

The methods presented herein use a gradient-based optimization algorithm (Excel Solver) that requires a smooth function. Typically, costs can be expressed as continuous, smooth functions, even when the real relationship between design parameters and cost is not smooth. For example, consider that the cost of excavation increases with depth in discrete intervals. That is, the cost per unit volume may be one value for depths between 0 to 3 m, another value for depths between 3 and 6 m, and so on. It is possible to adjust a smooth function (e.g. polynomial or power function) to represent the unit cost of excavations for any depth. Similarly, some variables may

accept only discrete values, such as the area of a steel section. In that case, the optimization is performed with continuous variables, and then the result is rounded to available, discrete values.

Another factor to note is that objective functions may not be convex. When the objective function is convex, only one global maximum or minimum exists, and the result should not depend on the initial estimate. Unfortunately, most performance functions in foundation design are nonlinear, even when the safety margin is used. The solution found by the algorithm will depend on the initial estimate provided by the user. Other authors have warned of this situation (Low 2007, Phoon 2008). The user must validate the result, perhaps by comparing the result with traditional methods. It is a good practice to test different initial estimates to verify the stability of the solution (See Appendix B).

5.3 Single Component Optimization

The simplest model assumes that the system is formed by a single component, and there is only one possible failure mode. This assumption is not very realistic for common structures, but it is appropriate to illustrate the framework. The results will also serve to determine the influence of multiple failure modes in the optimum reliability.

To illustrate the concepts behind URBEQ, consider a component with only two design variables, d_1 and d_2 . If the probability distributions of the random variables are known, it is possible to calculate the reliability index for each point of a grid in the design space and draw contours of equal reliability, as shown in Figure 5-1. In this example, the reliability increases with increasing values of d_1 and d_2 .

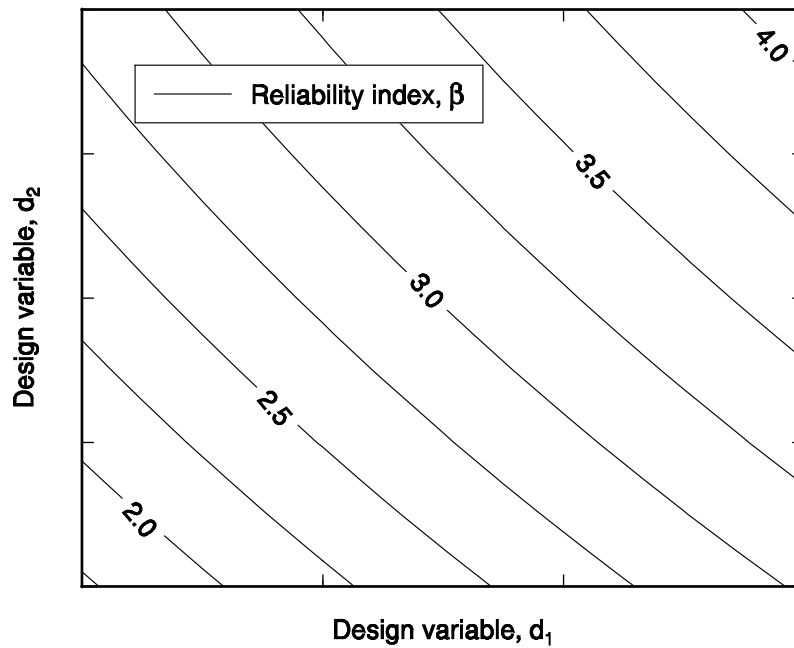


Figure 5-1. Reliability index contours in the design space

Each point also has an associated initial cost and a cost of failure that can be computed deterministically. The initial cost for each design point is shown in Figure 5-2.

The total expected cost can be computed for each design point using the reliability index, the initial cost, and the cost of failure according to Equation 5-1.

In general, for any number of design variables, there will be a hyper-surface similar to the surface represented by contour lines in Figure 5-3. The optimum design point is located at the lowest elevation of the hyper-surface. Of course, an efficient optimization method should be able to locate the optimum design point without having to calculate multiple expected costs.

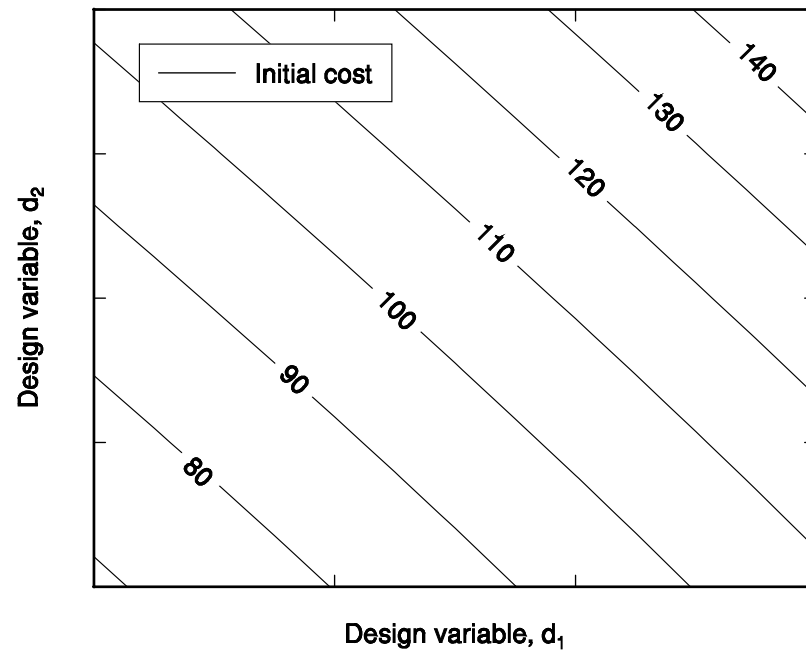


Figure 5-2. Initial cost contours in the design space

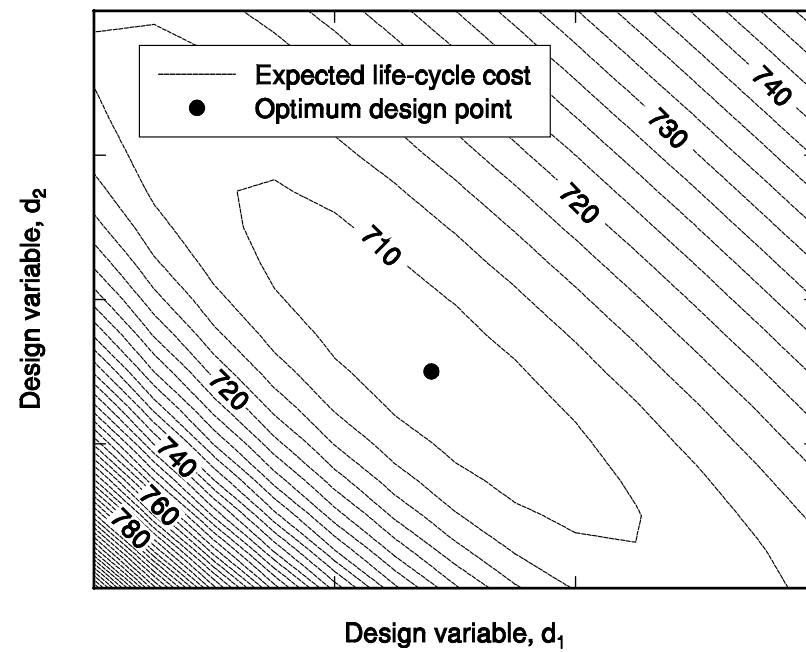


Figure 5-3. Expected life-cycle cost contours

5.3.1 Direct Monte Carlo Simulation

The first step in this approach is to create a number of simulations of all the random design parameters and random design variables according to their mean values and variances. Appendix C describes a procedure to generate random variables in Excel, including different probability distributions and correlated variables. The second step is to compute the performance function for each simulation and determine the number of failures with the Heaviside or unit step function, given below:

$$h_i = \begin{cases} 0 & \text{if } g_i(d, p) > 0 \\ 1 & \text{if } g_i(d, p) \leq 0 \end{cases} \quad (5-2)$$

in which $g(\cdot)$ = performance function, d = vector of design variables, and p = vector of random design parameters.

The probability of failure is simply the average value of the counting function, h , from all the simulations, as given below:

$$p_f = E(h) = \frac{1}{n} \sum_{i=1}^n h_i \quad (5-3)$$

In reality, the design variables, d , are also random, because small defects during construction will result in slightly different values from those specified. However, the variability typically is much smaller than the design parameters, p , so that they can be taken as deterministic. In a more detailed analysis, design variables can be random, and the optimization algorithm will search for their optimum mean values.

Note that the objective function, C_T , is not smooth. It has steps caused by the “if” statements of the Heaviside step function. As mentioned previously, gradient-based optimization algorithms can be used only with smooth functions. Currently, there are three solutions to this problem. The first is to use heuristic methods (e.g. genetic algorithms) to optimize the design. Unlike gradient-based methods, heuristic methods can find accurate solutions for non-smooth functions and can include discrete variables. Unfortunately, the computational effort is significantly larger, and the user has to adjust several parameters by trial and error to increase the efficiency and accuracy of the method. The second is to substitute the unit step with a smooth function to compute the number of failures. A readily available function is the standard normal distribution. However, for thousands or millions of simulations, it is preferable to use a less expensive option such as the logistic function shown in Figure 5-4.

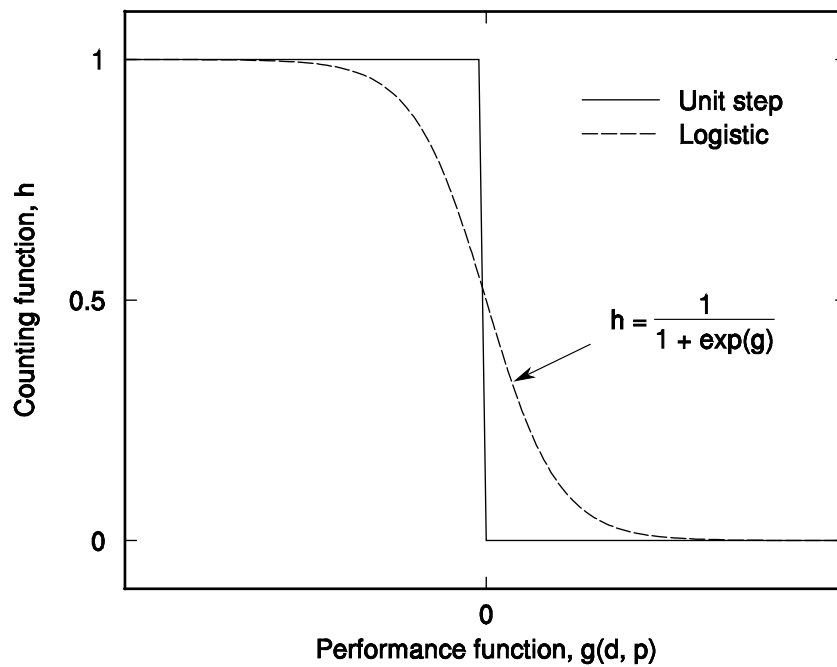


Figure 5-4. Counting functions for Monte Carlo simulation

The error introduced by the approximation can be calculated simply as the difference in the probability of failure calculated with the two methods.

The third solution is to use the response surface method (RSM), described in Chapter 3, to obtain an approximation of the probability of failure as a function of the design variables and parameters. The resulting function is smooth and can be used to solve the optimization problem. Gasser and Schuëller (1997) used this method in RBDO. They used a second-order polynomial to approximate two results: an implicit performance function (finite element analysis) and the probability of failure from MCS. If the performance function is explicit, only one response surface approximation is necessary. Gasser and Schuëller obtained a function to approximate the natural logarithm of the probability of failure, $\ln(p_f)$. It is not advisable to use p_f directly, because the values can span several orders of magnitude and the polynomial approximation will not be accurate. A better option is to approximate the reliability index, which varies over a small range, using the following:

$$\beta \approx a + \sum_{i=1}^n b_i d_i + \sum_{i=1}^n \sum_{j=1}^i c_{ij} d_i d_j \quad (5-4)$$

in which a , b , and c = regression coefficients, and d = design variables.

The number of points required to solve the linear system of equations, which is a function of the number of design variables, is shown below:

$$N = 1 + n + n(n + 1) / 2 \quad (5-5)$$

in which n = number of design variables.

There is no universal procedure to locate the design points for the interpolation. In general, it is recommended to select evenly spaced values, covering the entire feasible domain of each variable.

Once the coefficients of the polynomial approximation are known, the function is inserted into the cost objective function (Eq. 5-1), and the problem can be solved by any nonlinear gradient-based optimization algorithm.

5.3.2 First Order Reliability Method (FORM)

The FORM can also be used to solve the design optimization problem. In fact, this may be the most practical method given its flexibility, precision, and lower computational cost. The double loop problem in RBDO can be seen clearly when trying to use a spreadsheet for both FORM and design optimization calculations. Excel Solver is needed for both tasks. However, only one instance of Solver can be running at any given time, even if function calls are made from macros in Visual Basic. Certainly, there are other optimization algorithms available that can be called from custom-made computer codes, but one of the original objectives of this study is to use only a spreadsheet.

Shan and Wang (2008) showed a single-loop method for RBDO using the inverse most probable failure point (MPP) concept. In their paper, they computed the inverse MPP for a given target reliability index. Since it is necessary to compute the optimum reliability instead of prescribing a target, some modifications are required. If the reliability index is set as a design variable, and the following constraint is added to the problem, only one call to Solver is made.

$$\begin{cases} \text{minimize} & C_T(d, p) \\ \text{subject to} & g(d^*, p^*) = 0 \\ \text{by changing} & d, \beta \end{cases} \quad (5-6)$$

$$p_i^* = m_{pi} - \beta s_{pi}^2 \frac{\partial g / \partial m_{pi}}{\left(\sum_{i=1}^n (s_{pi} \partial g / \partial m_{pi})^2 \right)^{0.5}} \quad (5-7)$$

in which d^* and p^* = MPP design parameters and variables, m_p and s_p = mean and standard deviation of the random variables.

If the design variables, d , are considered random variables, the MPP (d^*) is also calculated using Equation 5-7. The probability of failure then is calculated as $p_f = \Phi(-\beta)$.

For correlated random variables, Basha and Babu (2008) showed how to calculate the design point, as given below:

$$\begin{aligned} p_k^* &= m_{pk} - \beta s_{pk} \sum_{i=1}^n \alpha_i \sqrt{\lambda_i} \Omega_{ki} \\ \alpha_k &= \frac{\sum_{i=1}^n \frac{\partial g}{\partial x_i} s_i \sqrt{\lambda_i} \Omega_{ik}}{\sqrt{\sum_{j=1}^n \left[\sum_{i=1}^n \frac{\partial g}{\partial x_i} \left(s_i \sqrt{\lambda_i} \Omega_{ij} \right) \right]^2}} \end{aligned} \quad (5-8)$$

in which λ and Ω = eigenvalues and eigenvectors of the correlation matrix, respectively.

Equation 5-8 is basically a transformation of the design variables to a set of equivalent uncorrelated variables. If the off-diagonal terms of the correlation matrix are zero, Equation 5-8 simplifies to Equation 5-7. Correlation of the design parameters affects the location of MPPs because the probability ellipsoids are distorted (Figure 5-5).

Consequently, the reliability index also is different. The probability of failure can be higher or lower in the correlated case, depending on the sign of the correlation and the influence of each variable in the performance function.

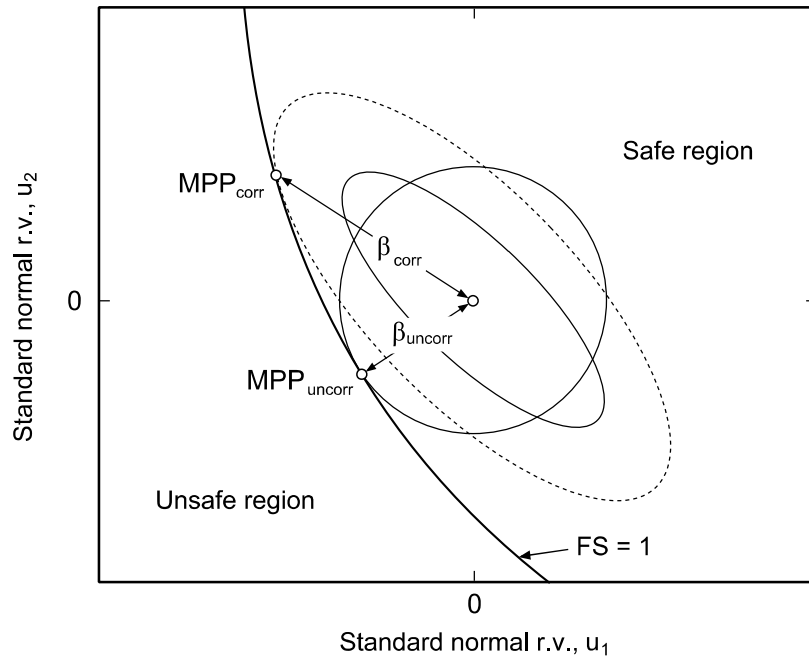


Figure 5-5. Change of reliability index for correlated variables

The MPP obtained with the inverse method is not exactly the same point found in direct FORM calculations. The difference is caused by one simplification of the method. Note that the partial derivatives in Equation 5-7 are evaluated at the mean value of the design parameters, p . This simplification is necessary, because the MPP is unknown. The algorithm will search the MPP in the direction indicated by the partial derivatives. If the contours of the performance function are parallel, the algorithm will find the exact MPP. In other cases, the solution will be slightly different from the direct FORM result, but this difference is usually within the approximation error (Figure 5-6).

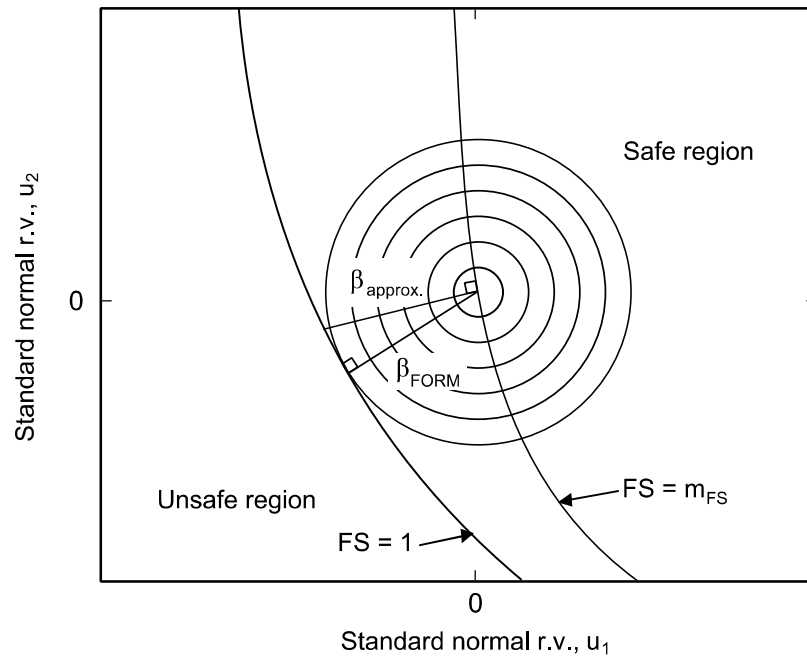


Figure 5-6. Different MPPs from direct and inverse FORM

When one or more variables are not normally distributed, the reliability index can be approximated with FORM using an upgraded version of the spreadsheet method presented by Low and Tang (2007). The method computes an equivalent normal mean and variance of the non-normal random variable, at the MPP, using the Rosenblatt transformation (see Appendix D).

Since FORM is already an approximation, it is necessary to estimate the error of the additional simplification. Figures 5-7 and 5-8 show a comparison of FORM and MCS results for non-normal random variables using the simplified safety margin equation. It is assumed that MCS results are “correct”, because they were computed from a large number of simulations using the corresponding distribution for each random variable. In contrast, FORM calculations require equivalent parameters for non-normal distributions.

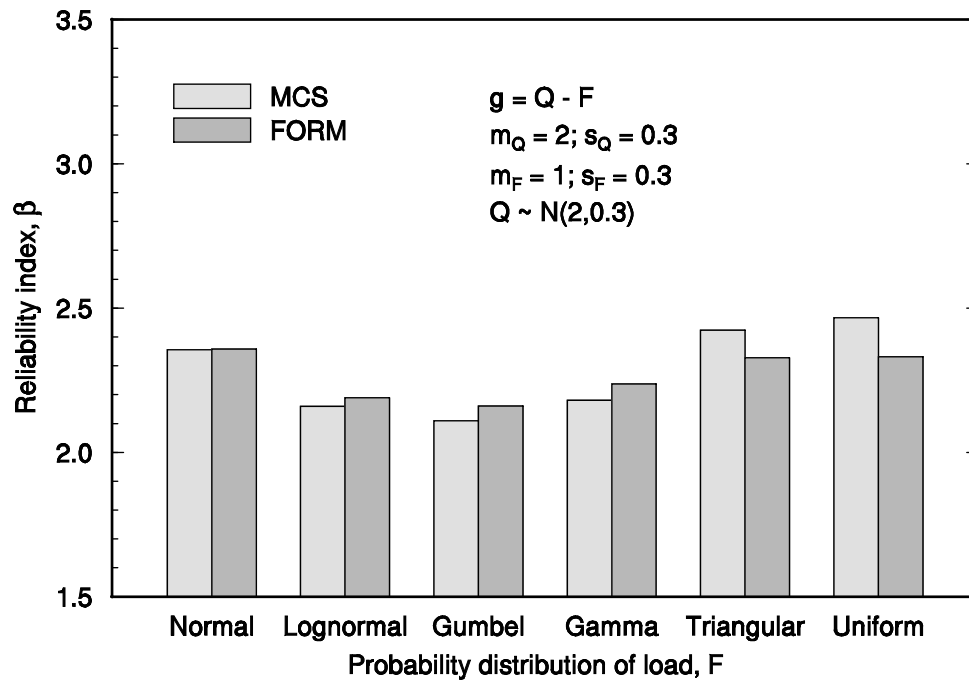


Figure 5-7. Reliability index for normal Q and various distributions of F

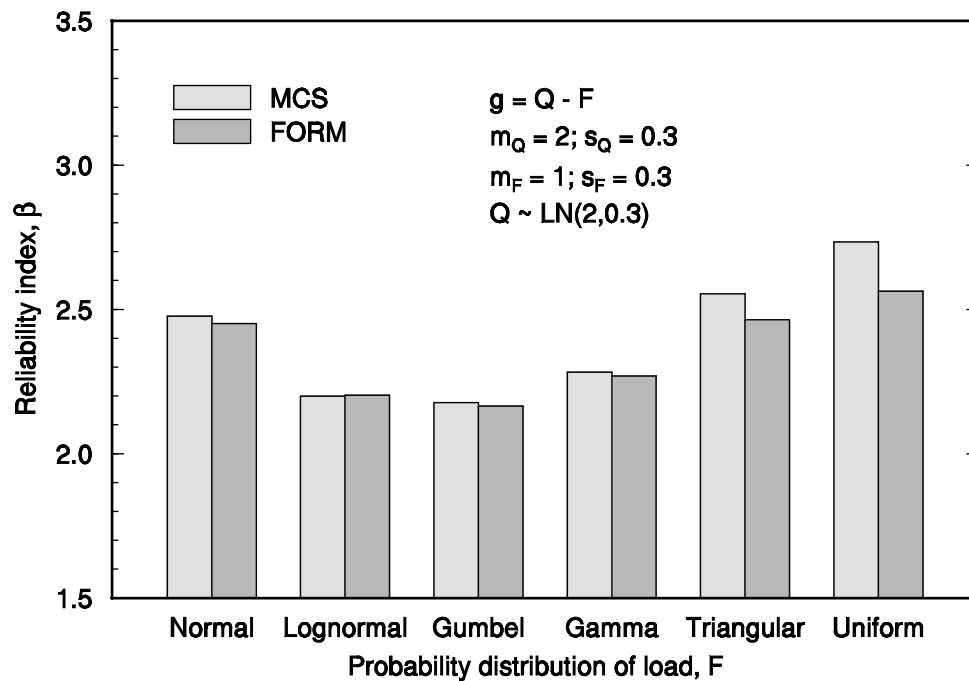


Figure 5-8. Reliability index for lognormal Q and various distributions of F

Apparently the approximation is not very accurate for triangular and uniform distributions, but the results are more accurate than simply assuming normal distributions for all random variables. Figure 5-8, in which lognormal distribution of the resistance is assumed, shows similar variability for different types of load distributions.

5.3.3 First Order Second Moment (FOSM)

RBDO problems can also be solved using the mean value FOSM to calculate the probability of failure. Although the method is straightforward, the results can have large errors, as discussed in Chapter 2. Unlike FORM or MCS, the result from FOSM is not invariant to the shape of the performance function. Therefore, only partial results are shown in the examples of this chapter for comparison purposes.

In FOSM, the reliability index of a structure is computed from the mean value and the standard deviation of the performance function as shown below:

$$m_g \approx g(m_p) \quad (5-9)$$

$$s_g^2 = \left[\frac{\partial g}{\partial m_p} \right]^T C \left[\frac{\partial g}{\partial m_p} \right] \quad (5-10)$$

in which C = covariance matrix.

The mean value of g is approximately equal to the function evaluated at the mean values of the design parameters, and the variance can be computed with Equation 5-10. The partial derivatives in the equation typically are replaced by finite differences centered on the mean values.

The correlation structure of the variables is included in the calculation of the variance. The distribution of the variables can not be specifically included, because the method is based on the first two statistical moments.

Hsu et al. (2007) proposed a method to determine the validity of FOSM results by calculating the most probable failure point (MPP). If the approximation is good, the MPP should lie near the failure surface ($FS = 1$ or $M = 0$). However, if the error is unacceptable, another method must be used. The examples presented herein show that the errors introduced by FOSM are considerably high and, generally, unacceptable.

5.4 Example 1: Spread Footing on Sand

To illustrate the methods explained in the previous section, two examples are shown. In both cases, the performance functions are explicit limit state equations used for typical foundation designs.

The first example is a simple square spread footing sitting on dry sand, considering only general shear bearing capacity failure. The solution will determine optimum values for two deterministic design variables: depth, D , and width, B , of the foundation. An illustration of the problem, along with the design parameters and cost assumptions, is shown in Figure 5-9.

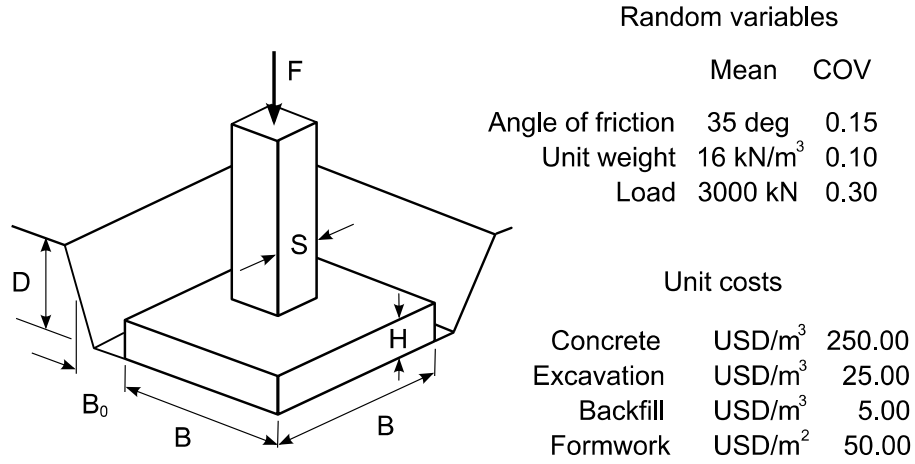


Figure 5-9. Illustration of spread footing and design parameters

The limit state equations using the factor of safety and the safety margin are shown below:

$$g = FS - 1 = \frac{B^2 q_{ult}}{F} - 1 \quad (5-11)$$

$$g = M = B^2 q_{ult} - F \quad (5-12)$$

in which B = foundation width, q_{ult} = ultimate bearing capacity, and F = load effect.

The bearing capacity is calculated as (Vesic 1975):

$$q_{ult} = 0.5 B \gamma N_\gamma \zeta_{\gamma s} \zeta_{\gamma d} + \gamma D N_q \zeta_{qs} \zeta_{qd} \quad (5-13)$$

in which D = depth of the foundation, γ = soil unit weight, N_γ and N_q = bearing capacity factors, and $\zeta_{\gamma s}$, $\zeta_{\gamma d}$, ζ_{qs} , ζ_{qd} = shape and depth modification factors.

The bearing capacity factors and their modification factors for depth and shape are given below:

$$\begin{aligned}
N_\gamma &= 2(N_q + 1) \tan \phi \\
N_q &= \exp(\pi \tan \phi) \tan^2(45 + \phi / 2) \\
\zeta_{\gamma s} &= 0.5 \quad \text{for square footings} \\
\zeta_{\gamma d} &= 1 \\
\zeta_{qs} &= 1 + \tan \phi \\
\zeta_{qd} &= 1 + 2 \tan \phi (1 - \sin \phi)^2 [\tan^{-1}(D / B)]
\end{aligned} \tag{5-14}$$

in which ϕ = angle of friction. Note that the inverse tangent function must be in radians.

The costs shown in Figure 5-9 include materials, equipment, and labor. Therefore, the initial cost is the product of the unit cost and the estimated volume. The cost of concrete includes an average cost of reinforcement per unit volume. The volumes for each item are calculated as follows:

$$\begin{aligned}
V_{\text{exc}} &= D(B + 2B_0)^2 \\
V_{\text{conc}} &= HB^2 + S^2(D - H) \\
V_{\text{fill}} &= (D - H)(B + 2B_0)^2 \\
A_{\text{fw}} &= 4BH + 4S(D - H)
\end{aligned} \tag{5-15}$$

in which B_0 = overexcavation distance, H = footing thickness, S = pedestal width, and A_{fw} = area of formwork.

Although only variables B and D are optimized, other variables need not be constant. For a more realistic model, H is taken as 15% of the footing width, B_0 is 25% of the excavation depth, and S is equal to 0.3 m.

The optimization process using MCS is relatively straightforward. First, all the design parameters (i.e. material properties, ground conditions, and loads) and construction costs must be defined. Then, the probability of failure is computed from all the realizations of the random variables. The total expected cost is calculated for the initial estimates of B and D. Lastly, Solver is used as follows: minimize the total expected cost, C_T , by changing the values of B and D, subject to the constraint $FS = 1$ (or $M = 0$). The calculations can take a few minutes depending on the computing power, number of random variables, and the number of realizations.

In FORM optimization, calculations are much faster. The spreadsheet used for this example is shown in Figure 5-10. The initial cost was calculated using the unit costs defined for each material and the volumes (Eq. 5-15) calculated for the initial values of B and D. The initial design point is calculated with Equation 5-7 using an initial estimate of the reliability index, β' . In cell G25, the safety factor is calculated using the bearing capacity equations and the initial design point. Cell H25 contains a matrix operation to calculate the nominal reliability index, β : $=SQRT(MMULT(G21:I21-TRANSPOSE(D8:D10),MMULT(MINVERSE(B21:D23),TRANSPOSE(G21:I21)-D8:D10)))$. For matrix operations in Excel, it is necessary to press ctrl, shift, and enter keys simultaneously after the equation is typed in the cell. Now Solver can find the minimum value of cell H17 (C_T) by changing H8, H9, and I25 (B, D, and β'), subject to the constraint $G25 = 1$ ($FS = 1$).

	A	B	C	D	E	F	G	H	I	J
1										
2		Reliability-Based Foundation Design Optimization								
3										
4		Instructions: Edit the necessary cells with the appropriate values. Click "Optimize" to run the optimization code.								
5										
6		Soil properties					Optimum Design			
7		Name	Units	Mean	COV		Name	Value		
8		Angle of friction	deg	35.0	0.15		Depth, D	2.4835		
9		Unit weight	kN/m3	16.0	0.1		Width, B	3.0999		
10		Max axial load	kN	3,000	0.3		Thickness, H	0.4650		
11							L/B	1		
12		Costs					Pedestal, S	0.3		
13		Name	Units	Unit Cost			Overexc, B0	0.62087		
14		Concrete	m3	\$200.00			Initial Cost	\$2,699.94		
15		Excavation	m3	\$25.00			Exp. fail cost	\$465.60		
16		Fill	m3	\$5.00			Expected cost	\$3,165.54		
17		Formwork	m2	\$50.00						
18		Cost of failure		\$996,000.00						
19							Design Point			
20		Covariance matrix					ϕ	γ	F	
21		27.5625	0	0			19.019	15.343	4109	
22		0	2.56	0						
23		0	0	810000						
24							$FS(d^*)$	β	β'	pf
25							1.0000	3.3094	3.3094	0.000467
26										
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Figure 5-10. Spreadsheet for Example 1 using FORM

Similarly, FOSM was used in a spreadsheet for Example 1. In FOSM, Solver can find the optimum design with the following parameters: minimize C_T , by changing B and D. In this case, there are no constraints in the optimization algorithm. Unfortunately, FOSM results have the largest errors of the three methods discussed herein.

5.4.1 Results for Nominal Reliabilities

Results from MCS, FORM, and FOSM optimization using the safety factor equation (Equation 5-11) are shown in Figure 5-11 for different normalized costs of failure.

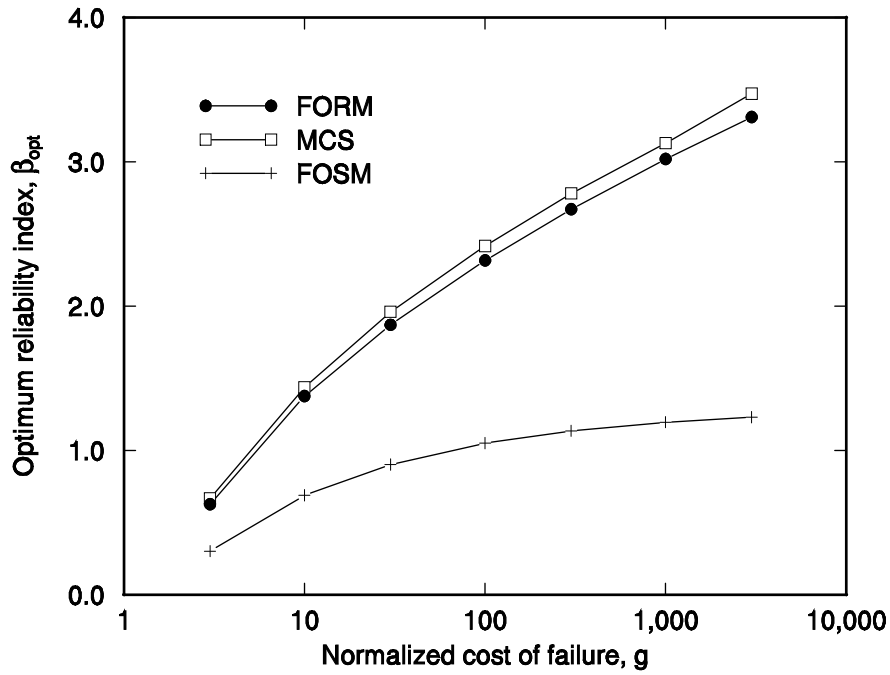


Figure 5-11. Optimum reliability vs. cost of failure

The normalized cost of failure, g , is the ratio of the cost of failure over the initial cost when $FS = 1$ ($g = C_F/C_0$), as defined in Section 3.4.1. For this example, C_0 is USD332.00.

The results from MCS and FORM are relatively consistent for all the values of the normalized failure cost. On the other hand, the results using FOSM are considerably lower, because of the assumptions in its formulation. If the safety margin is used as the performance function, FOSM results show slight improvements, but the method is still inappropriate for RBDO.

Optimization analyses provide optimum reliabilities without cost approximations and the optimum set of design variables. The optimum values for depth, D , and width, B , are shown in Figures 5-12 and 5-13. These optimum variables also result from minimizing the objective function using Excel Solver.

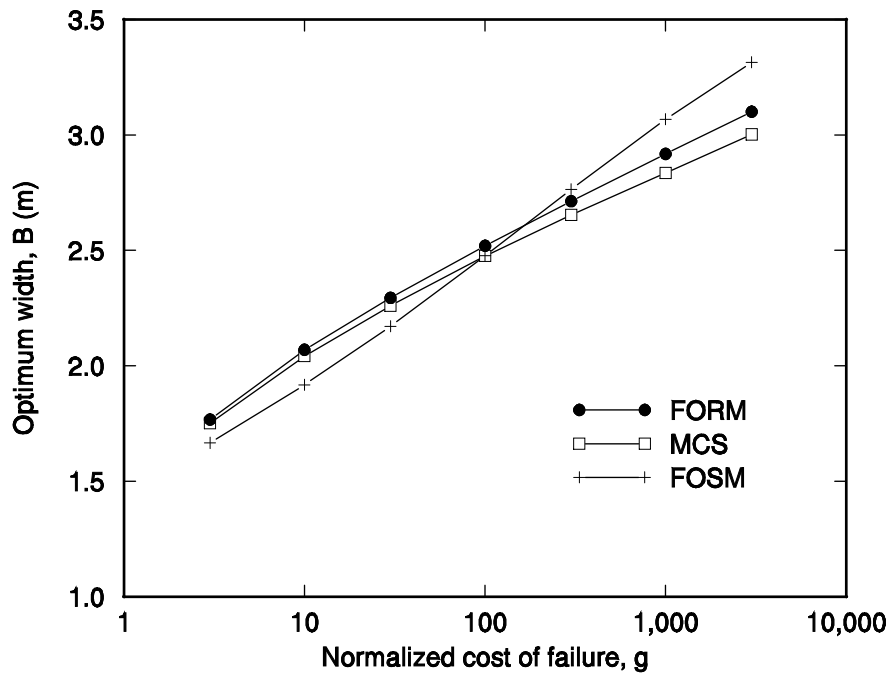


Figure 5-12. Optimum width vs. cost of failure

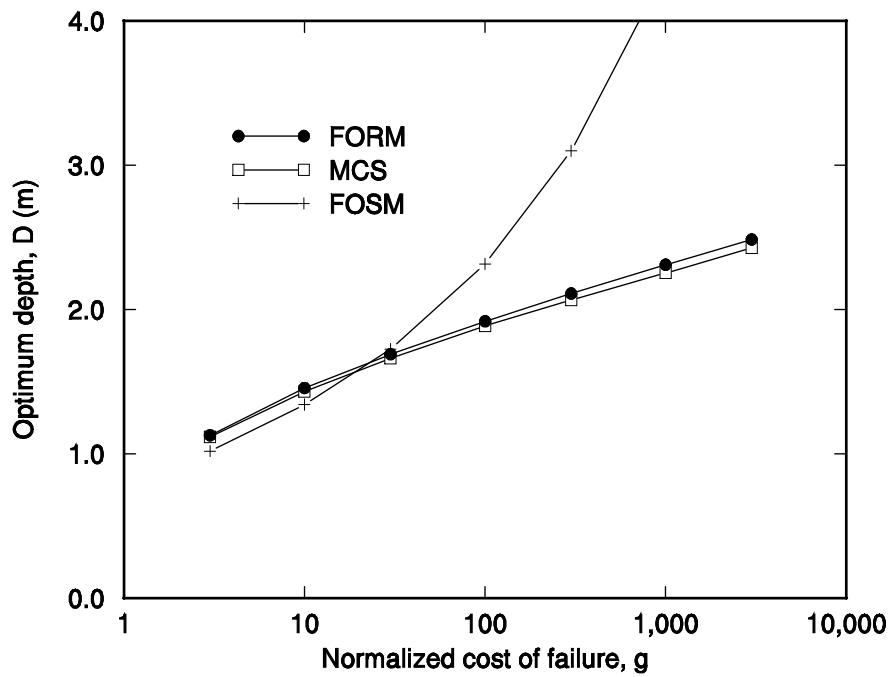


Figure 5-13. Optimum depth vs. cost of failure

To compare these results with the closed-form solutions shown in Chapter 3, it is necessary to estimate the variability of a lumped resistance term and the constants of the linear cost function. Kanda and Ellingwood (1991) stated that the normalized cost ratio, k , varies between 0.05 and 0.1 for typical building structures. In this example, k is approximately 0.75. This value can be estimated by calculating the initial cost for any safety factor and solving for k in Equation 3-5. The initial cost for $FS = 2.85$ is USD811.66 and $k = 0.78$. For $FS = 5.5$, the initial cost is USD1462.05 and $k = 0.75$.

The COV of the resistance, Q , for the parameters shown in Figure 5-9 is approximately 0.77. This result was obtained using MCS in a spreadsheet, in which Q is equal to the ultimate bearing capacity times the area of the foundation. The COV is simply the standard deviation of Q divided by its mean. It is possible to compute the standard deviation from a large number of simulations of the random parameters (ϕ and γ) as described in Section 5.3.1.

Using the previous results and the closed-form equations presented in Chapter 3, optimum reliability indices were computed for different values of g . The results are shown in Figure 5-14.

Note that the normal and Gumbel closed-form solutions yield higher values of optimum reliabilities. This difference is expected since the Gumbel solution assumes a deterministic resistance, while the solution for normal distributions assumes that COV_Q is similar to COV_F . The lognormal solution shows better agreement with numerical optimization results using FORM.

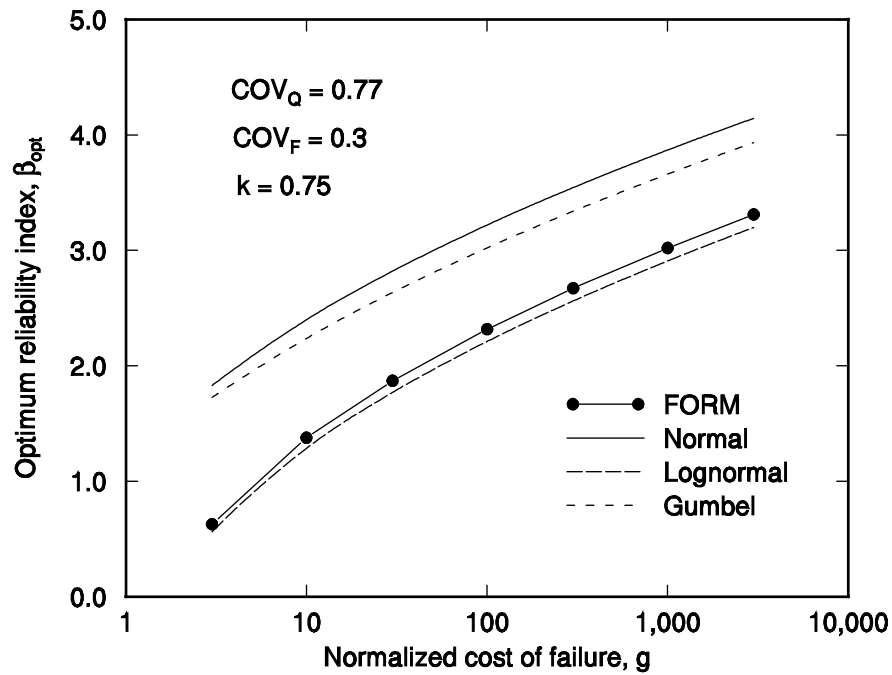


Figure 5-14. Comparison of closed-form solutions and LCCO with FORM

A better comparison is shown in Figure 5-15, where all closed-form solutions and FORM consider that the resistance term, Q , is deterministic ($COV_Q = 0$). There is better agreement between all solutions. However, it is not realistic to assume that resistance is deterministic. The approximate solutions for normal and Gumbel distributions should not be used, because the error can be large when the COV of the resistance is large.

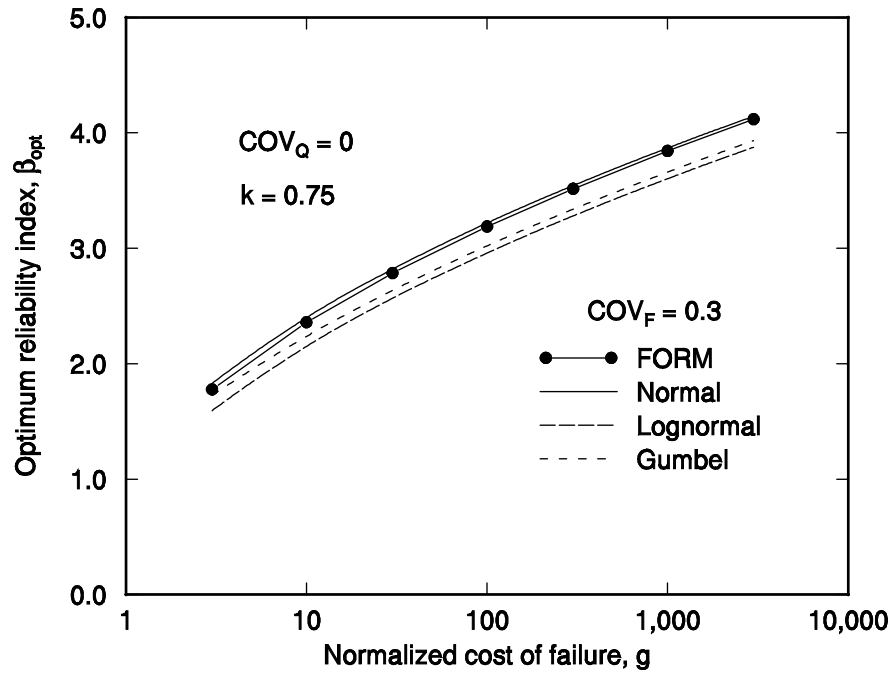


Figure 5-15. Comparison of closed-form solutions and FORM for $COV_Q = 0$

The agreement between FORM and MCS is remarkable, considering that the limit state equation used was the safety factor. It is apparent that the error introduced in FORM by the location of the MPP is small enough to be acceptable. However, it is possible to calculate the difference between the direct and inverse approaches of FORM for this example. The reliability index from the direct approach is calculated using the spreadsheet method described by Low and Tang (1997) with the optimum depth and width obtained from the optimization procedure for each value of g . The results are shown in Table 5-1.

Table 5-1. Error introduced by inverse FORM – Example 1

g	β inverse	β direct	Error β	Error p_f
3	0.6266	0.6266	0.00%	0.00%
10	1.3758	1.3755	0.02%	0.05%
30	1.8693	1.8686	0.04%	0.17%
100	2.3157	2.3143	0.06%	0.38%
300	2.6702	2.6679	0.09%	0.67%
1000	3.0187	3.0153	0.11%	1.11%
3000	3.3094	3.3048	0.14%	1.62%

It is obvious that the error increases as the reliability index increases. This behavior is expected, because the MPP and the mean values of the design parameters are further apart when the reliability is high. In addition, it can explain the increasing difference between FORM and MCS as the normalized failure cost increases. Unfortunately, these results can not be extrapolated to other problems, because the error also depends on the shape of the performance function. However, in this example, the error in the probability of failure is less than 2%, even for the worst case, which is acceptable in the author's opinion.

Results shown for Example 1 do not consider human errors in their calculations. Nominal probabilities of failure were used to compute the expected cost of failure. A more accurate approach should consider extrinsic factors in the variability of the performance.

5.4.2 Results including Human Errors

According to the method called HERV, a coefficient of variation because of errors can be estimated with Equation 4-7. Since most foundations are designed for ULS target

reliability indices between 2.8 and 3.5 (p_f between 0.02% and 0.25%), and the true probability of failure is about one order of magnitude larger, the true reliability indices likely are between 1.5 and 2.5. These numbers result in a coefficient of variation because of errors between 0.28 and 0.56. Now, the performance function using the safety factor approach and the safety margin are given as:

$$g = FS - 1 + e = \frac{B^2 q_{ult}}{F} - 1 + e \quad (5-16)$$

$$g = M + e = B^2 q_{ult} - F + e \quad (5-17)$$

in which e = error term, F = load term, and M = safety margin.

As mentioned previously, the mean value of the error is zero, and the standard deviation is calculated from an average performance function and COV of the error. In this example, it is assumed that the average safety factor is equal to three.

Nominal and true reliability indices were calculated using the same procedure described in the previous section, except that Equation 5-16 was used as the performance function. Here, it is assumed that $COV_e = 0.5$. Results are shown in Figure 5-16.

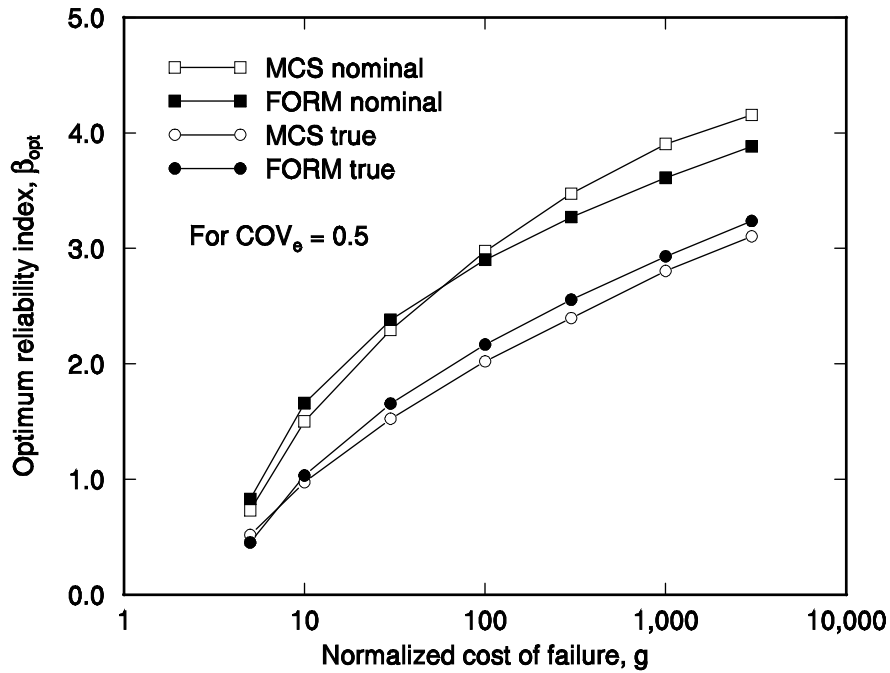


Figure 5-16. Nominal and true reliability indices

True reliability indices from MCS and FORM show good agreement for all the values of g and are very similar to the results from the analysis without human errors. However, these results are true values, while the previous results were nominal. The optimum nominal reliability indices, including human errors are computed as follows. In MCS, the objective function, C_T , considers the true probability of failure, but another column in the spreadsheet contains the original performance function (without error terms). The nominal probability of failure is the number of times that g is less than zero divided by the total number of realizations. Then the nominal reliability index is calculated with the inverse standard normal probability function ($\beta = \Phi^{-1}(p_f)$).

Calculating nominal reliability indices with FORM is not straightforward, because the MPP changes when errors are disregarded. An equivalent nominal reliability can be obtained in a spreadsheet with the procedure proposed by Low and Tang (1997), described in Section 2.3.1, using the optimum values of the design variables (B and D)

and without the error term. The resulting value can be interpreted as the optimum nominal reliability index when human errors are included. However, the MPPs located with the inverse approach and the regular FORM do not coincide, because human errors are not considered in the latter. In any case, it is clear that the nominal reliability index increases with increasing human error COV while the true value decreases.

It is important that the optimum design variables obtained with the proposed approach be similar to the values obtained with MCS. Figure 5-17 shows the values for the optimum width and depth from FORM and MCS.

Figure 5-18 shows the optimum nominal and true reliability indices for different values of COV_e calculated using the same proposed FORM optimization procedure.

These results are in agreement with the observed behavior of target values. When the variability of errors increases, the difference between nominal and true values also increases. Results from optimization can be compared also with the closed-form solution presented in Section 4-4. The closed-form solution uses a different approach to include the effect of error. It uses the ratio of the nominal and true reliability indices, v .

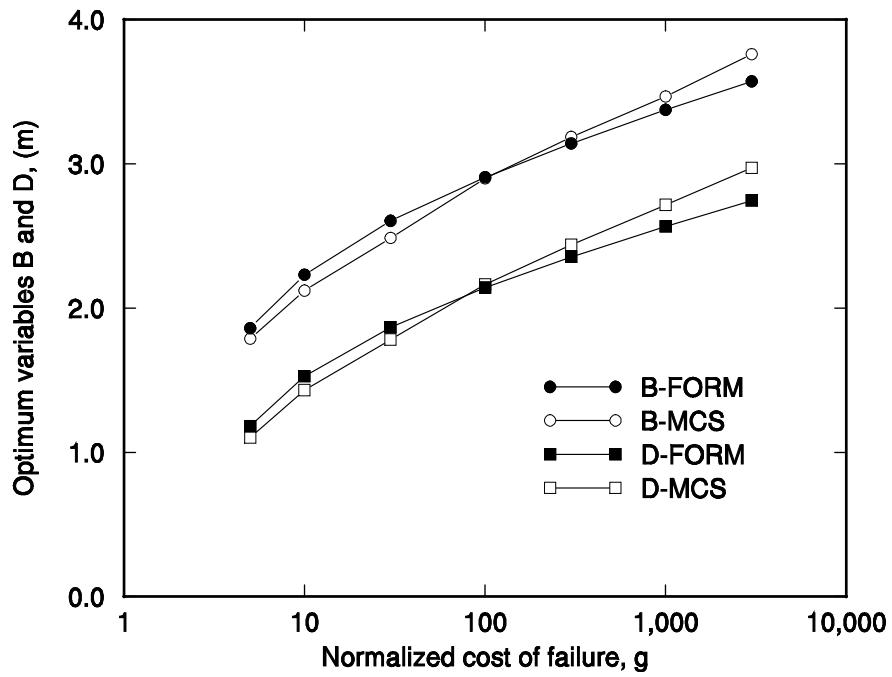


Figure 5-17. Optimum width and depth including errors

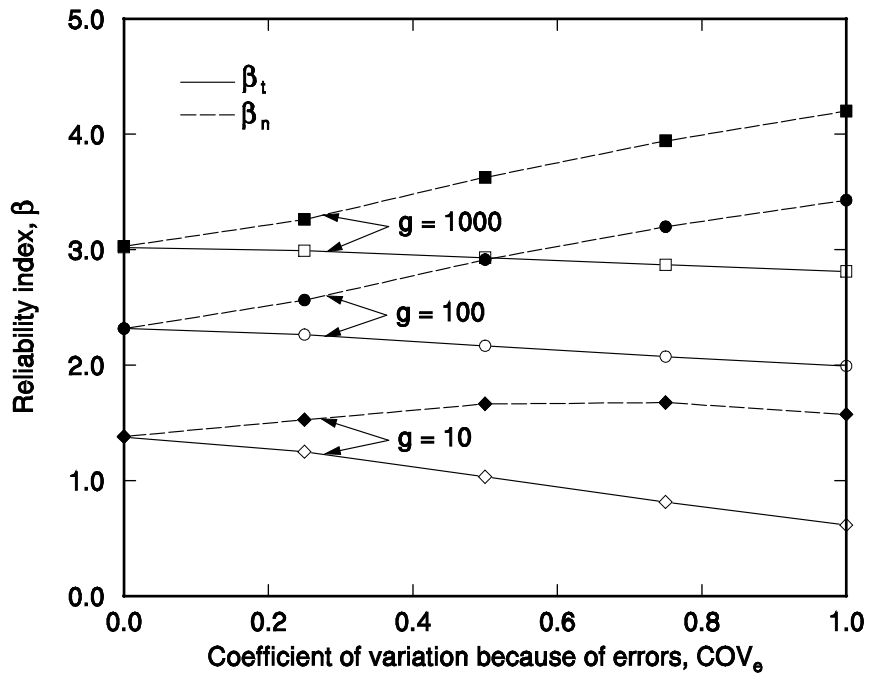


Figure 5-18. Optimum reliability as a function of COV_e

However, the agreement between the two approaches is remarkably good, considering that one is a single equation with five parameters and the other is a full optimization analysis (Figure 5-19).

The other method proposed in Chapter 4 to consider human errors is the error ratio. This approach can be used also with FORM or MCS procedures. The only difference between this method and the nominal optimization is that the probability of failure is calculated using the true reliability index ($\beta_t = \beta_n/v$).

Figure 5-20 shows the variation of optimum nominal and true reliability indices for Example 1.

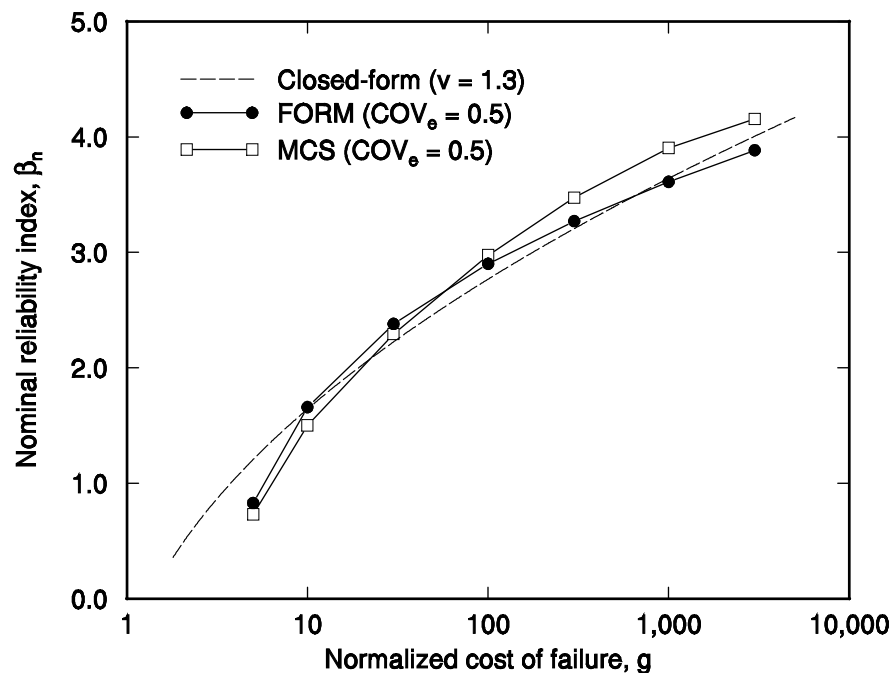


Figure 5-19. Comparison of optimum nominal reliability indices including errors

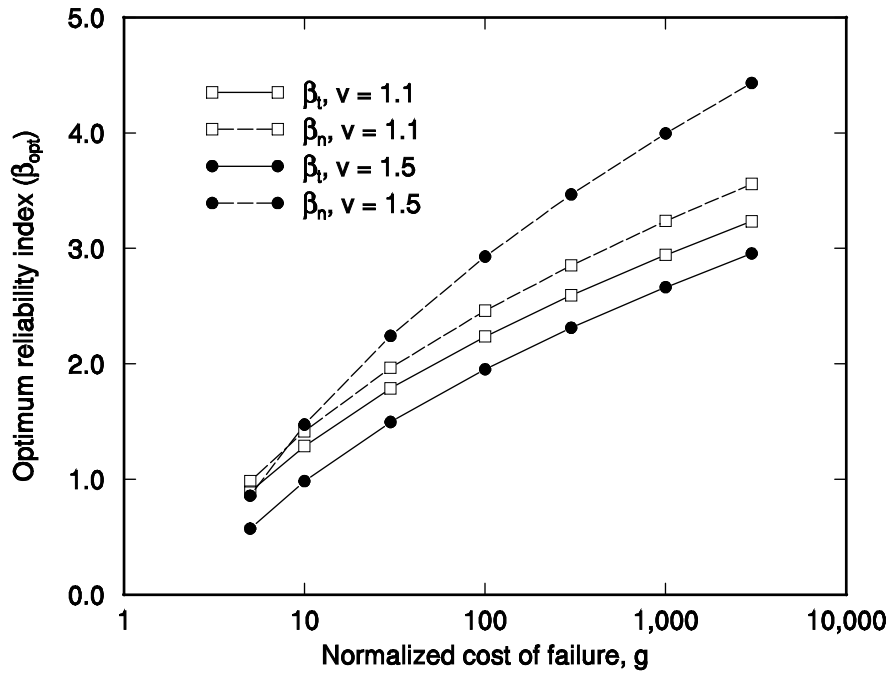


Figure 5-20. Optimum reliability indices for two error ratios, v

5.4.3 Sensitivity Analysis

As in any function, some parameters and design variables affect the result more than others over the entire range of possible values. A sensitivity analysis for Example 1 shows that the COVs of the soil unit weight, the load term, and the error term have small influence on the optimum reliability index. On the other hand, the COV of the friction angle and the normalized failure cost have the largest influence on the result. The sensitivity of the true reliability index is shown in Figure 5-21, and the sensitivity of the total expected cost is shown in Figure 5-22.

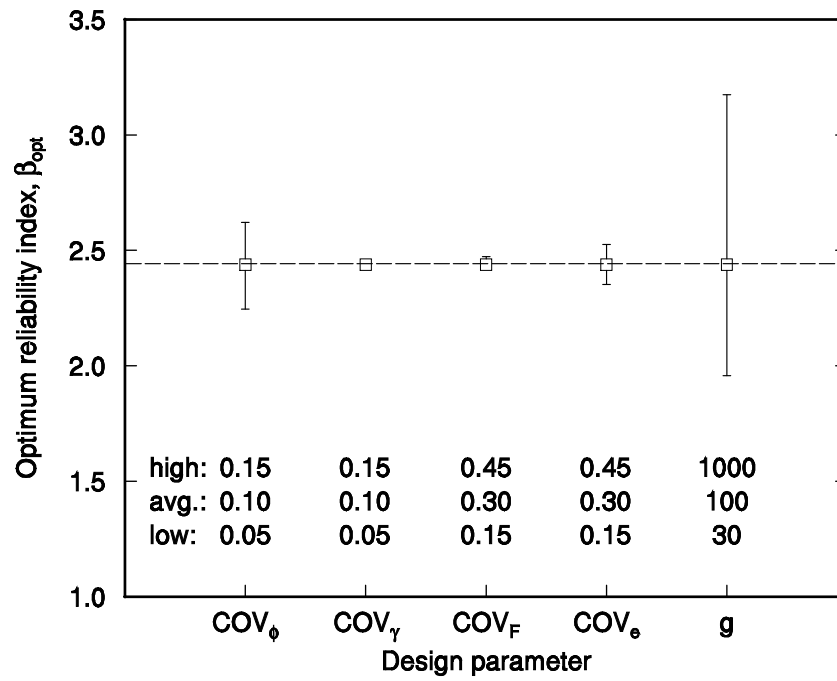


Figure 5-21. Sensitivity of the true reliability index – Example 1

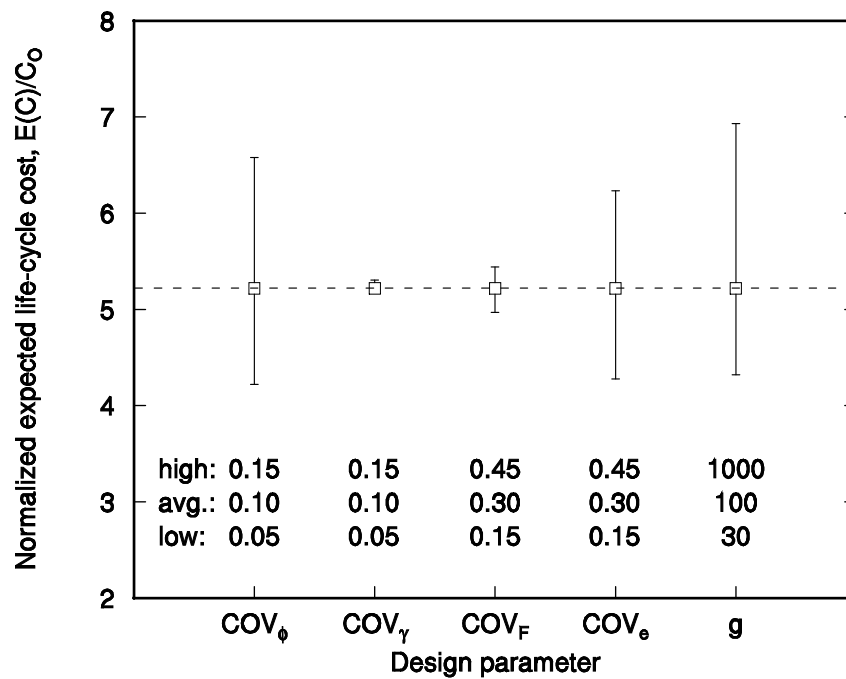


Figure 5-22. Sensitivity of expected life-cycle cost – Example 1

It is known that the soil unit weight has small variability compared to other parameters. Besides, the friction term of the bearing capacity equation is directly proportional to the unit weight. The large influence of the friction angle was expected, because the bearing capacity factors N_c , N_γ , and N_q , vary significantly with small changes in ϕ . The influence of the normalized failure cost is caused simply because its value can vary over several orders of magnitude.

5.5 Example 2: Drilled Shaft in Clay

Example 2 is the design of a simple drilled shaft in clay for an axial load. The variables, statistical properties, and costs are shown in Figure 5-23.

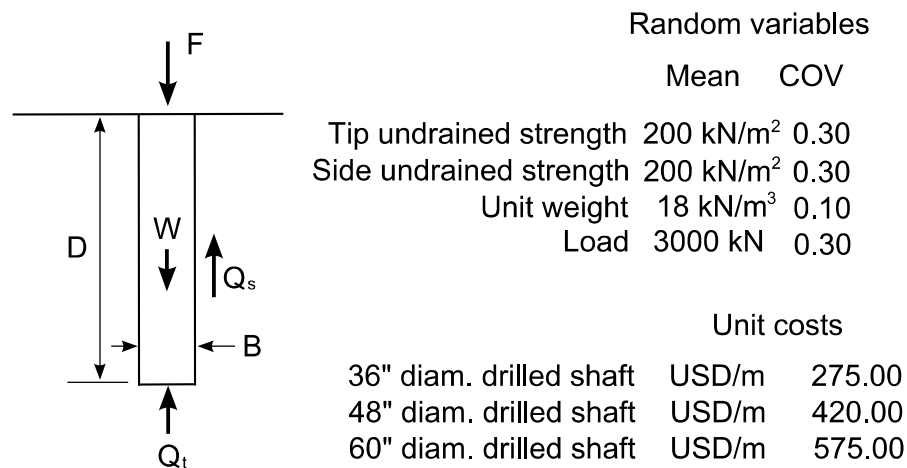


Figure 5-23. Illustration of a drilled shaft with design parameters

In addition to the cost per unit length of drilled shaft, typically there is a fixed cost for mobilization of equipment. In this example, a fixed cost of USD500.00, USD700.00, or USD900.00 was considered for 0.9 m, 1.2, and 1.5 m diameter shafts, respectively.

The capacity of the shaft is equal to the sum of the tip and side resistances minus the self-weight. Similar to Example 1, the safety factor or the safety margin can be used as performance functions and are given below:

$$g = FS - 1 = \frac{Q_s + Q_t - W}{F} \quad (5-18)$$

$$g = M = Q_s + Q_t - W - F \quad (5-19)$$

The side and tip resistances were calculated using the following equations:

$$Q_s = \pi B D \alpha_n s_u \quad (5-20)$$

$$Q_t = (5.14 s_u \zeta_{cs} \zeta_{cd} + \gamma D \zeta_{\gamma s} \zeta_{\gamma d}) 0.25 \pi B^2 \quad (5-21)$$

in which α_n = adhesion factor and ζ_{ij} = modification factors.

The adhesion factor was determined according to Chen and Kulhawy (1994) as:

$$\alpha_n = 0.31 + 0.17 p_a / s_u + \varepsilon \quad (5-22)$$

in which p_a = atmospheric pressure (≈ 100 kPa) and ε = random variable associated with the regression.

Both modification factors for the surcharge term are equal to 1.0 because the problem assumes undrained conditions. The shape factor for the cohesion term is equal to 1.2, and the depth factor is a function of B and D, as given below:

$$\zeta_{cd} = 1 + 0.33 \tan^{-1} \left(\frac{D}{B} \right) \quad (5-23)$$

in which $\tan^{-1}(\cdot)$ = inverse tangent function in radians.

As mentioned previously, it is possible to fit an equation to compute the unit cost of construction as a function of the shaft diameter. However, in this example, it is easier and more accurate to run the optimization process for each available shaft diameter. Therefore, the only design variable in each optimization is the shaft depth, D .

5.5.1 Results for Nominal Values

Following the same procedure used in Example 1, the optimum reliability index and optimum depth for each shaft diameter were computed using FORM and MCS.

FORM optimization uses the inverse MPP calculation and Excel Solver to find the minimum expected cost by changing the design variables (optimum depth, D). Similarly MCS uses Solver to find the minimum expected cost, but the probability of failure is calculated with multiple simulations of the random variables.

Optimum reliability levels are somewhat different, because each shaft diameter has a different initial cost when $FS = 1$ (C_0). Therefore, a particular value of the normalized failure cost, g , does not correspond to the same failure cost, C_F , for the three diameters. The initial cost, C_0 , for each shaft diameter is given below:

- For $B = 0.9$ m, $C_0 = \text{USD}2800.00$
- For $B = 1.2$ m, $C_0 = \text{USD}2187.00$
- For $B = 1.5$ m, $C_0 = \text{USD}1503.00$

Closed-form results for optimum reliability, using lognormal load and resistance terms, were calculated to compare them with FORM results. The value of k was taken

as 1.5, because the initial cost for $B = 1.2$ m and $FS = 3$ is $C_I = 700 + (420)(23.7) = \text{USD}10696.00$. Using Equation 3-5, the value of k is equal to 1.4. However, for other shaft diameters and safety factors, the value of k can be larger. The results for each shaft diameter using FORM and the closed-form solution are shown in Figure 5-24.

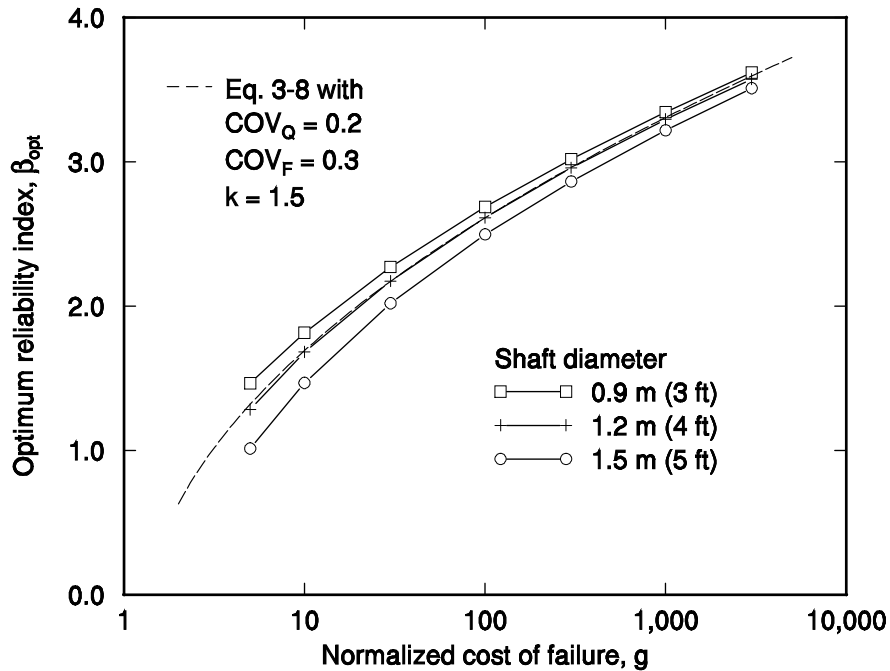


Figure 5-24. Optimum reliability indices using FORM and closed-form solution

Optimum depths from FORM and MCS are shown in Figure 5-25. Both methods give similar results and are reasonable, because the optimum depth increases as the cost of failure increases for the three shaft diameters.

Using the same procedure demonstrated in Example 1, the error introduced in the inverse FORM was calculated for drilled shafts with $B = 1.2$ m and is shown in Table 5-2.

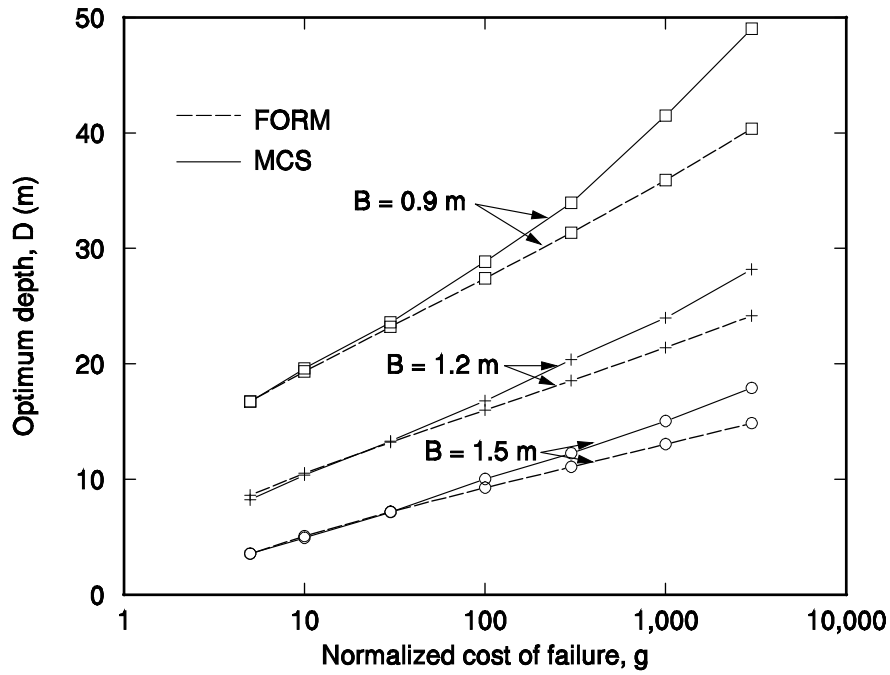


Figure 5-25. Optimum depths for each shaft diameter

Table 5-2. Error introduced by inverse FORM – Example 2 (safety factor)

g	$\beta_{\text{opt}} \text{ inverse}$	$\beta_{\text{opt}} \text{ direct}$	Error β_{opt}	Error p_f
3	0.8900	0.8809	1.04%	1.30%
10	1.6707	1.6146	3.48%	10.93%
30	2.1613	2.0431	5.79%	25.27%
100	2.5975	2.3965	8.39%	43.26%
300	2.9401	2.6534	10.81%	58.83%
1000	3.2743	2.8847	13.51%	72.97%
3000	3.5512	3.0614	16.00%	82.59%

Errors in Example 2 are much higher than in Example 1. This difference is caused by the shape of the performance function. One way to reduce the error, which is about one order of magnitude for the probability of failure, is to use the safety margin as the performance function instead of the safety factor. Note that the partial derivatives of the safety margin equation are not functions of any design parameter. Therefore, the

reliability from the inverse and the direct FORM must be identical. Numerical results for Example 2 using the safety margin are shown in Table 5-3 and Figure 5-26.

Table 5-3. Error introduced by inverse FORM – Example 2 ($B = 1.2$ m, safety margin)

g	β_{opt} inverse	β_{opt} direct	Error β_{opt}	Error p_f
3	0.8623	0.8623	0.00%	0.00%
10	1.6098	1.6098	0.00%	0.00%
30	2.0717	2.0717	0.00%	0.00%
100	2.4735	2.4735	0.00%	0.00%
300	2.7802	2.7802	0.00%	0.00%
1000	3.0685	3.0685	0.00%	0.00%
3000	3.2955	3.2955	0.00%	0.00%

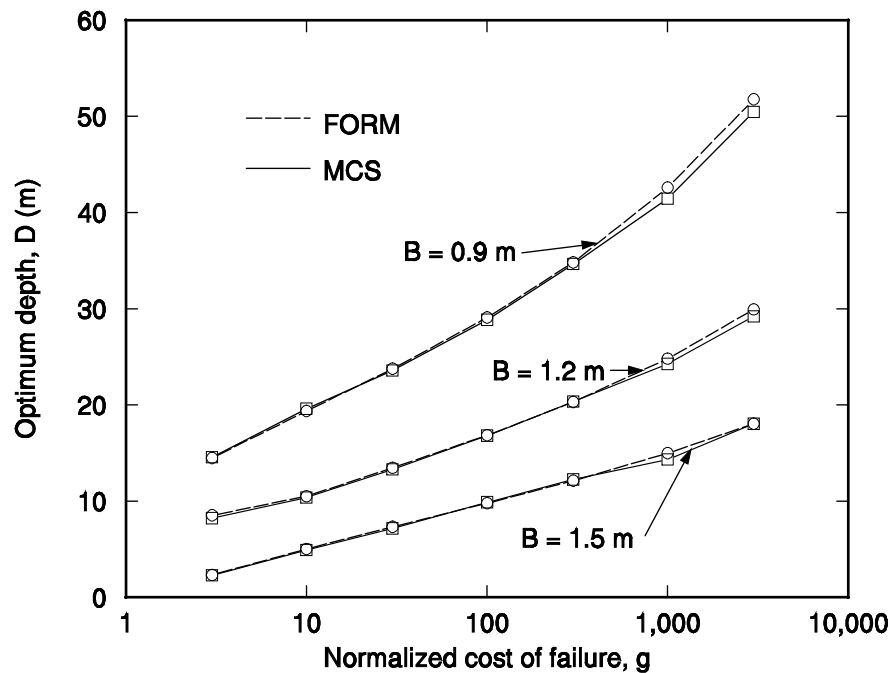


Figure 5-26. Optimum shaft depth using the safety margin equation

The FORM results improve substantially using the safety margin instead of the safety factor. Clearly, the differences between FORM and MCS results were caused by the

shape of the performance function. Typically, the safety margin is better for reliability approximations, because the parameter, M , is a linear function of the load and resistance terms. In this case, the error resulted in smaller shaft depths and higher optimum reliability indices, but in other cases, the error can be acceptable.

5.5.2 Results Including Human Errors

In the previous section, it was shown that the safety margin works better as the performance function in FORM optimization, because the error introduced by the inverse calculation is zero. However, this section presents optimization results including human errors using the safety factor equation to observe the behavior of the method when human error is not zero.

Using a COV because of errors equal to 0.5, the true reliability indices decrease (Figure 5-27), and the optimum depths for each shaft diameter increase (Figure 5-28). The closed-form solution including errors (Eq. 3-8) is not shown here, because the reliability indices in the figure are true values, and the equation was derived for nominal values.

Results in this example are similar to Example 1, where optimum reliability curves move downwards, but nominal reliabilities increase. In general, the shape of optimum depth curves is different in FORM and MCS results. For some values of g , the optimum design variable (D) is higher with FORM, but the opposite occurs for higher values of g .

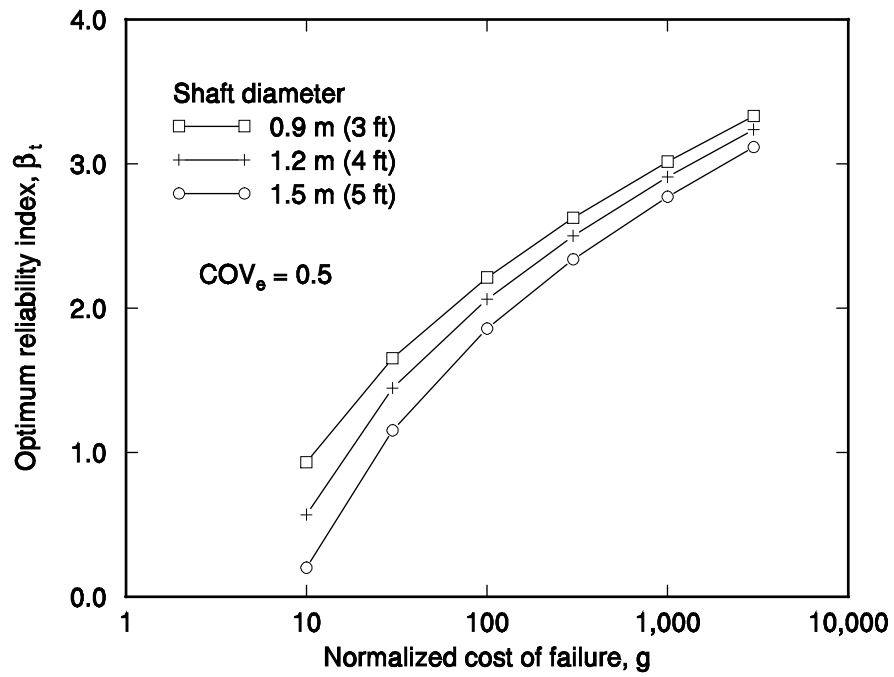


Figure 5-27. Optimum true reliability index including human errors

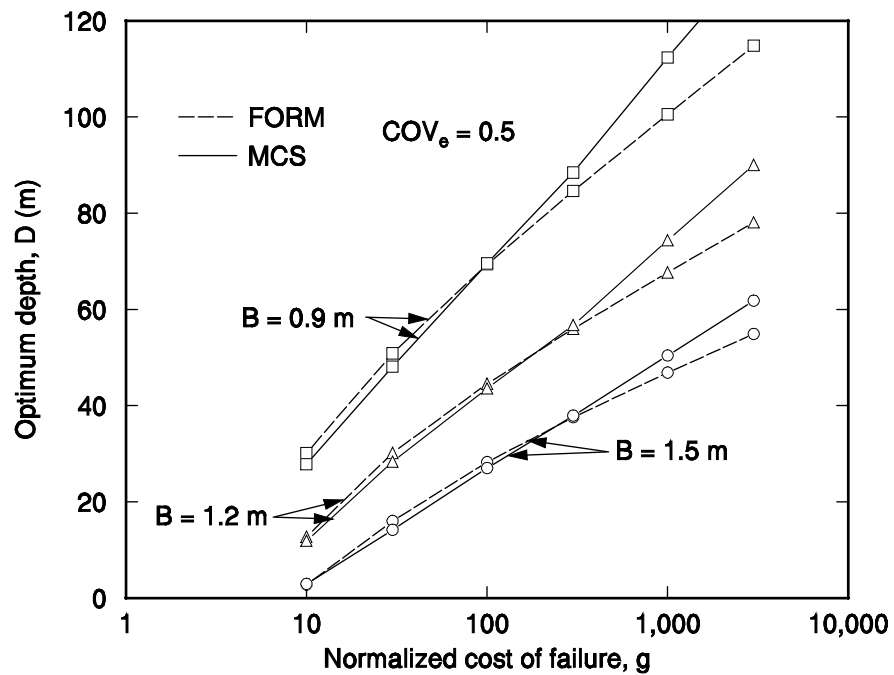


Figure 5-28. Optimum depth using FORM including human errors

5.5.3 Sensitivity Analysis

For a sensitivity analysis of Example 2, the random variables considered are: undrained shear strength at the tip of the shaft, s_{ut} ; undrained shear strength on the sides, s_{us} ; unit weight of soil, γ ; the load term, F ; and the error term, e . For this analysis, mean values are used. Upper and lower values are shown next to the results in Figure 5-29. The variation in expected life-cycle cost is shown in Figure 5-30.

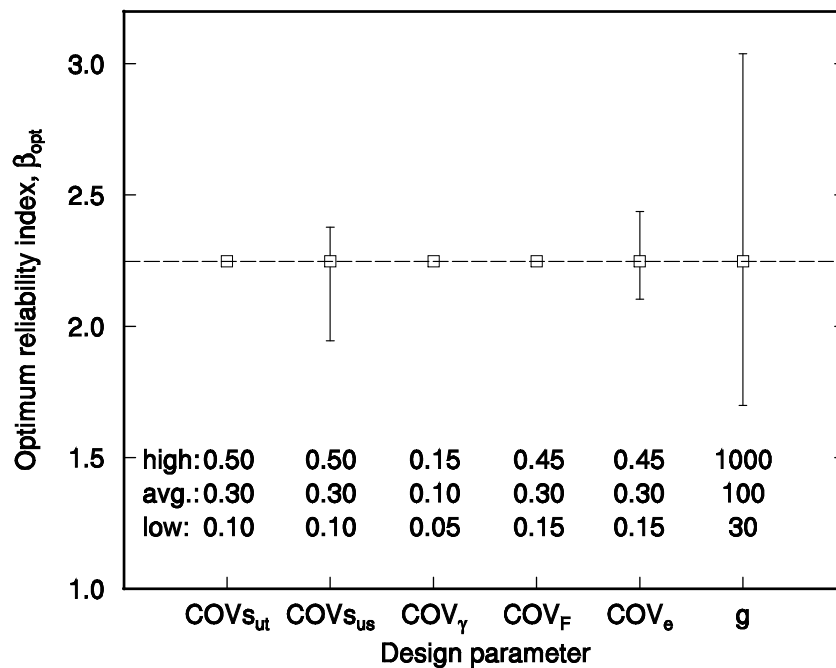


Figure 5-29. Sensitivity of the true reliability index – Example 2

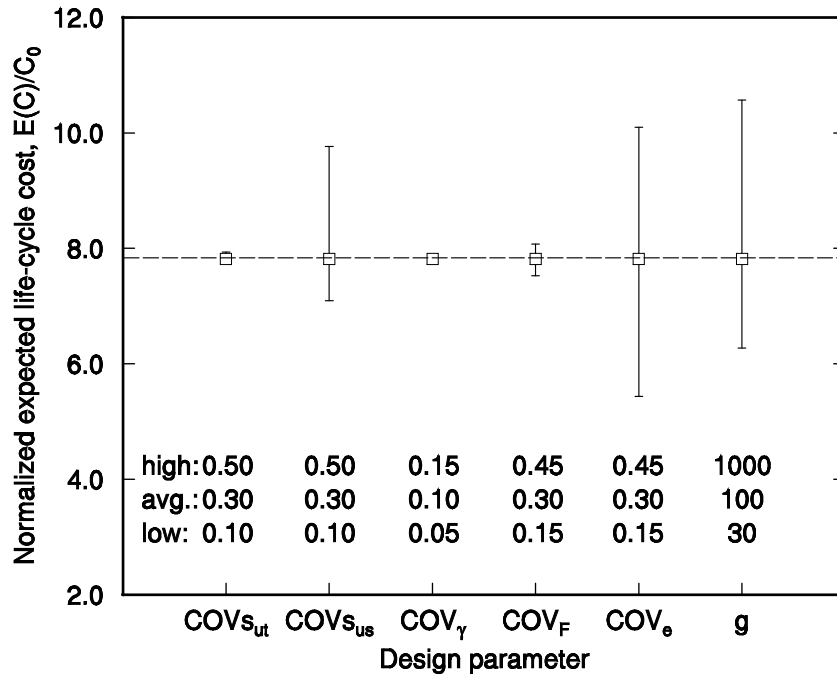


Figure 5-30. Sensitivity of expected life-cycle cost – Example 2

5.6 Multiple Component Optimization

In the examples of the previous sections, it was assumed that only one failure mode was present and the system had a single component. In reality, structures and their foundations are complex systems with multiple components and multiple failure modes. It is known that the reliability of a series system with identical components is lower than the reliability of the individual components. Grigoriu and Turkstra (1979) presented the following equation to calculate the reliability of a system with correlated components in series:

$$p_{\text{sys}} = \int_{-\infty}^{\infty} \left[\Phi \left(\frac{\beta_e + \sqrt{\rho} y}{\sqrt{1-\rho}} \right) \right]^n \phi(y) dy \quad (5-24)$$

in which β_e = reliability index of each component, ρ = correlation coefficient between components, and n = number of components.

Figure 5-31 shows the ratio of system to component reliability indices as a function of the correlation coefficient between components. As the number of components increases, the reliability of the system decreases. Similarly, the reliability of the system decreases for lower values of the correlation coefficient. These theoretical results are useful to understand the effect of correlation and number of components on the reliability of the system. However, real structures are not simple combinations of elements in series or parallel. There are interactions between components that add another level of complexity to the optimization problem. Consider the case of a footing located on top of an undetected loose seam. If the structure is isostatic, loads can not be redistributed to other elements, and the footing will exhibit excessive deformations when loads are applied.

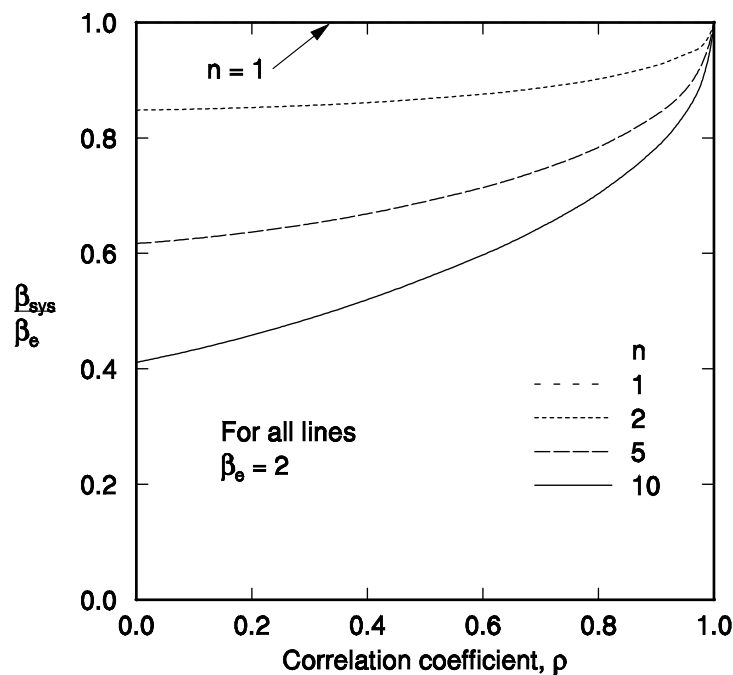


Figure 5-31. System reliability index for n correlated components in series

However, if the structure is hyperstatic or contains redundant elements, loads can be redistributed to other footings so that settlements can be considered acceptable.

Consider a very simple model of a building represented as a system with only two components - superstructure and substructure. If each component has only one failure mode, then only four states are possible during the life of the system: failure of the superstructure, failure of the substructure, failure of both components, and no failure.

Unfortunately, time-invariant reliability models are not very realistic, because when only one component fails, the system may be decommissioned or repaired. In both cases, the components are modified and their reliability changes. The intersection of two failures would occur only when both components fail simultaneously. More realistic predictions require the use of time dependent models, which rapidly increase the complexity of the problem.

Assuming that the probability of simultaneous failure is very small and that the components are repaired or decommissioned after failure, it is possible to use an approximate model in which failure events are disjoint (i.e. mutually exclusive).

In many cases, an ultimate limit state failure of the superstructure will have similar consequences as a foundation failure. For this scenario, the expected total cost can be estimated as:

$$E(C_T) = C_{I1} + C_{I2} + C_F(p_{f1} + p_{f2}) \quad (5-25)$$

in which C_{Ii} and p_{fi} = initial cost and probability of failure of the i^{th} component, respectively.

Typically, foundation designers do not have control over the reliability of the superstructure, since the design process of the two components is done separately. This situation is not ideal, because the performance of both subsystems is closely related, as discussed previously. However, for practical purposes, geotechnical engineers may have to design foundations for predetermined superstructure designs. To minimize the total expected cost with respect to the safety factor of the foundation, the derivative of Equation 5-25 must be set equal to zero. Assuming that initial costs of the two subsystems are given by the linear cost function shown in Equation 3-5, the derivative with respect to the safety factor of the i^{th} component is:

$$\frac{dE(C_T)}{dFS_i} = C_{0i} k_i + g C_{0i} \frac{dp_{fi}}{dFS_i} = 0 \quad (5-26)$$

in which C_0 = initial cost for $FS = 1$, g = normalized failure cost = C_F/C_0 , and k = normalized initial cost ratio.

Note that for these conditions, the optimum design of the foundation does not depend on the reliability of the superstructure. The optimum reliability of the foundation can be obtained with any of the methods discussed in this chapter.

Some numerical examples, using the closed-form solution for lognormal load and resistance, are shown in Figures 5-32 and 5-33. The values of the parameters selected for these examples are not representative of any type of structure. They were chosen simply as possible combinations of variability and cost parameters, but other values can be used. In Figure 5-32, the COV_Q of the superstructure is lower than the COV_Q of the substructure, but in Figure 5-33 both values are identical. The values of k also are different in each example.

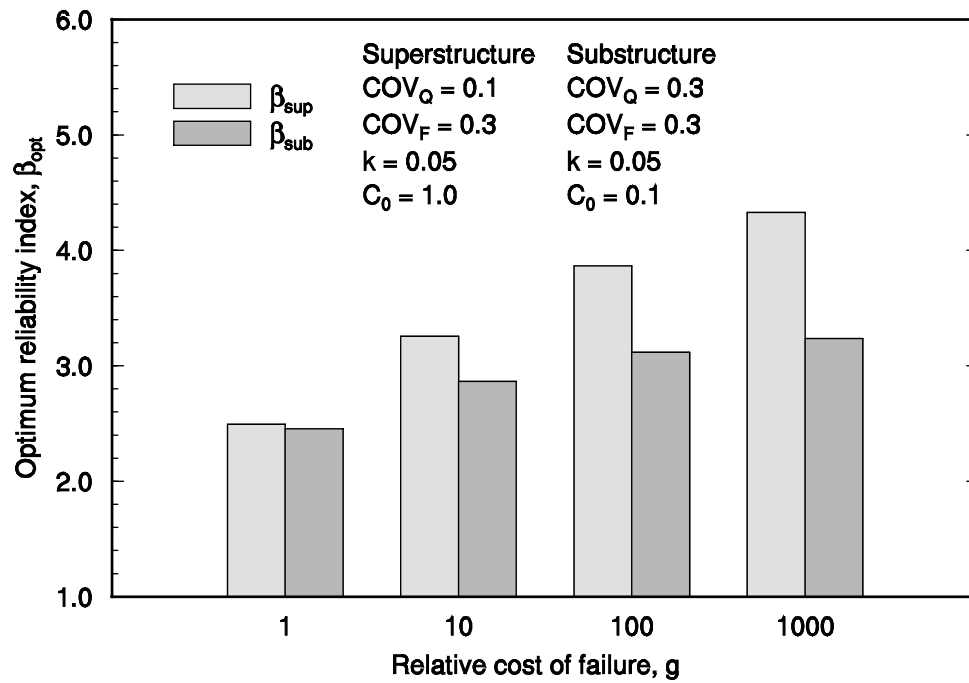


Figure 5-32. Optimum reliability indices for two components – Example 1

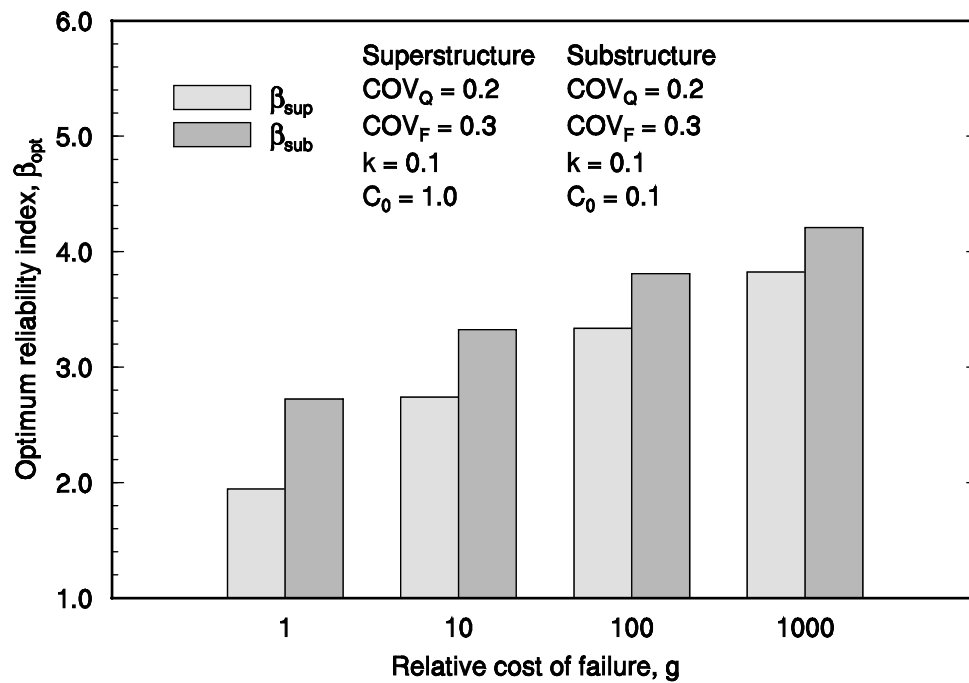


Figure 5-33. Optimum reliability indices for two components – Example 2

The design optimization of a simplified two-component system shows very interesting results. First, results from closed-form equations are identical to the results from FORM optimization, confirming that the optimum reliability of the foundation does not depend on the reliability of the superstructure. Secondly, there are cases in which the optimum reliability of the substructure can be less than the optimum reliability of the superstructure. This conclusion may seem counterintuitive, because a foundation failure causes the loss of the system in most cases. However, in some projects, the cost of higher reliability of one component can be prohibitive. When the performance of a component is a function of highly variable parameters, such as the resistance of some geomaterials, it may be preferable to accept higher risk in one of the components even if the consequences of failure are the same.

5.7 Summary

It has been shown that the optimum reliability level for a structure can be calculated using the life-cycle cost minimization approach stated in Chapter 3. Cost optimization approaches must consider the true probability of failure in their formulations, not nominal values. For this reason, two methods to include human errors, proposed in Chapter 4, were implemented in the optimization framework.

Two complete and efficient optimization methods, without linear cost assumptions, were described using FORM and MCS techniques. The results from two foundation design examples show that these methods can achieve similar results and are comparable with a closed-form solution that assumes lognormal load and resistance variables (Figures 5-19 and 5-24).

There are a number of advantages of full optimization techniques over closed-form solutions. For example, the results offer an optimum value for each design variable in addition to the optimum probability of failure. It is possible to use directly any continuous probability distribution for each random variable and consider valid correlation structures. Initial costs can be expressed as any smooth function of the design variables and parameters, not only as a linear function of the safety factor. It is not necessary to estimate the means and standard deviations of the load and resistance terms. Finally, it is possible to obtain both the true and nominal optimum reliability indices.

Some performance equations may introduce an error in reliability calculations using inverse FORM. In Example 1, using the safety factor equation introduced a small error that is considered acceptable. On the other hand, Example 2 showed a large difference between FORM and MCS calculations, and it was necessary to use the safety margin to reduce the error. It is necessary to investigate the effect of errors for each performance function to determine if FORM optimization is accurate.

Another important result from the examples shown is that some parameters may be more relevant than others for the optimum reliability index. A sensitivity analysis for the spread footing example showed that the variability of the friction angle, the normalized failure cost, and the variability of the error have the largest influence on the optimum reliability index.

Finally, the role of component reliability was considered in simple systems. In a simple model with two components, the optimum reliability of the foundation does not depend on the reliability of the superstructure. Furthermore, the optimum reliability of the foundation can be smaller or larger than the reliability of the superstructure

depending on the variability of the load and resistance, the cost of failure, and the initial cost of the components.

In general, the unconstrained reliability-based economic optimization (URBEO) problem can be solved rationally, using a simple spreadsheet with nonlinear optimization capabilities. The method using FORM is considerably simpler and more efficient than MCS. The most difficult aspect in the proposed framework probably is the selection of adequate human error parameters. Also, some knowledge of probability theory and reliability methods is necessary to use these techniques. In Chapter 6, a simplified method is presented to solve URBEO problems without full optimization analyses.

CHAPTER 6

SIMPLIFIED COST OPTIMIZATION

6.1 Introduction

Just as the balance between cost and safety is imperative in geotechnical design, there is a necessary balance between simplicity and accuracy to achieve useful and efficient design methods. In general, oversimplified models can not take into account all of the factors that affect the behavior of structures. On the other hand, excessively complex procedures may not be used by most practitioners or, even worse, they can be used incorrectly.

Reliability-based design methods have been available for many years, but LRFD and ASD still are used in most common design projects because they are less complex. As shown for the two examples presented in Chapter 5, the cost of failure is the most important parameter to determine the optimum reliability level. It is essential to provide practitioners with a simplified method that incorporates consequences of failure into the design.

Typically, coefficients for LRFD equations are determined through calibration procedures to achieve a target nominal reliability index. Some studies have shown that a single factor for the resistance term is insufficient to achieve uniform reliabilities (Phoon and Kulhawy 2002). MRFD, which provides multiple factors for the resistance equations, is a better approach to achieve target safety levels, especially in geotechnical design. A logical extension of MRFD to include costs of failure would determine different sets of resistance factors for a small number of importance

categories. Each category could have a fixed target reliability index, and the calibration would be carried out as usual.

However, the optimum reliability index is also a function of the load and resistance variations. This relationship increases the difficulty of a simplified approach, but it prevents a fundamental problem. In some cases, when the variability of the load and resistance is too high, no design can achieve the predetermined target reliability, even when very large safety factors are used. In addition, the MRFD format would require a very large number of factors, because at least five parameters are needed to determine the optimum safety level.

A simplified method for life-cycle cost optimization requires a different strategy, because the goal is not to achieve uniform nominal reliabilities, but optimum values. This chapter presents a new method to include costs in the design process.

6.2 Optimum Safety Levels

The proposed life-cycle optimization framework shown in the previous chapter gives an optimum design point for the assumed conditions and costs. Therefore, the optimum true probability of failure is only an intermediate result needed to compute the expected cost of failure. In the traditional LRFD method, designers do not select a target reliability to be used. Instead, they select an appropriate equation based on physical and statistical properties of the loads and materials considered. However, optimum safety levels are needed for calibration of LRFD equations or other simplified approaches.

If the probability of human errors is included, two intermediate results can be defined: the true reliability index and the nominal reliability index. These two values are related according to the models presented in Chapter 4.

Using the closed-form solutions from Chapter 4, the optimum nominal reliability index will depend on the load and resistance COVs, the initial cost parameters (C_0 and k), and the normalized cost of failure, g . If the true optimum reliability index is sought, an error parameter also is needed, either COV_e or the error ratio, v .

6.3 Optimum Safety Factors

When the simplified safety margin equation ($M = Q - F$) is used in probabilistic analysis, at least five parameters are needed to calculate the optimum nominal reliability index according to decision theory. The first step of the process is to calculate the reliability index for the possible range of values of each parameter. If low and high values are defined for each parameter, there are 32 possible combinations. These low and high values for each parameter are arbitrary, but they can be defined consistently with values presented in the literature and typical construction costs.

In the following example, it is assumed that the normalized cost of failure, g , ranges from 50 to 200 for typical structures. There are high-risk projects for which the cost of failure is greater than 200, but they usually have additional measures to mitigate risk, such as redundant or ductile elements. The range of k was taken from 0.5 to 1.5, according to the results of the two examples in Chapter 5. The COV of the resistance in geotechnical problems is generally higher than in other problems. The variability of loads depends on their nature and the combinations considered. Therefore, the ranges

of COV_Q and COV_F were 0.2 to 0.4 and 0.1 to 0.3, respectively. Finally, the error ratio, v , was estimated from 1.1 to 1.3, according to observed failure rates.

For each combination, the optimum nominal reliability index and the corresponding global safety factor are calculated using Equations 4-14 and 2-7. These two equations assume that the load and resistance terms have lognormal distributions. The results for the 32 combinations are shown in Table 6-1.

Note that the average reliability index is 3.16, which is consistent with values used for LRFD calibration of ULS equations. The average safety factor is 3.29, which is somewhat higher than traditional design values (2 to 3). The reason for this difference may be that designers always use conservative estimates of the load and resistance. When design loads and capacities are calculated according to building codes, real safety factors should be higher than estimated values.

While the average values show reasonable agreement with traditional approaches, the minimum and maximum reliability indices (2.25 and 4.26, respectively) are different from the traditional range. Also, the optimum safety factors vary from 1.91 to 5.82, which is a rather large range for traditional methods. These results indicate that the traditional safety levels may be adequate for average conditions, but they are not near optimum in all cases.

Designers can estimate the normalized cost of failure ($g = C_F/C_0$) from the importance of the structure and the initial cost when the safety factor is equal to one. The cost coefficient, k , can be determined for any design by calculating the initial cost for a given safety factor and solving for k using Equation 3-5.

Table 6-1. Optimum reliability indices for high and low parameter values

g	k	COV _Q	COV _F	v	β_n	FS
50	0.5	0.2	0.1	1.1	3.25	2.09
50	0.5	0.2	0.1	1.3	3.72	2.31
50	0.5	0.2	0.3	1.1	2.95	2.78
50	0.5	0.2	0.3	1.3	3.34	3.19
50	0.5	0.4	0.1	1.1	2.83	3.31
50	0.5	0.4	0.1	1.3	3.19	3.81
50	0.5	0.4	0.3	1.1	2.69	3.79
50	0.5	0.4	0.3	1.3	3.00	4.41
50	1.5	0.2	0.1	1.1	2.85	1.91
50	1.5	0.2	0.1	1.3	3.24	2.08
50	1.5	0.2	0.3	1.1	2.54	2.40
50	1.5	0.2	0.3	1.3	2.84	2.67
50	1.5	0.4	0.1	1.1	2.40	2.79
50	1.5	0.4	0.1	1.3	2.67	3.10
50	1.5	0.4	0.3	1.1	2.25	3.07
50	1.5	0.4	0.3	1.3	2.48	3.43
200	0.5	0.2	0.1	1.1	3.70	2.30
200	0.5	0.2	0.1	1.3	4.26	2.61
200	0.5	0.2	0.3	1.1	3.42	3.28
200	0.5	0.2	0.3	1.3	3.90	3.88
200	0.5	0.4	0.1	1.1	3.31	3.99
200	0.5	0.4	0.1	1.3	3.75	4.77
200	0.5	0.4	0.3	1.1	3.16	4.78
200	0.5	0.4	0.3	1.3	3.57	5.82
200	1.5	0.2	0.1	1.1	3.35	2.13
200	1.5	0.2	0.1	1.3	3.84	2.38
200	1.5	0.2	0.3	1.1	3.06	2.88
200	1.5	0.2	0.3	1.3	3.46	3.33
200	1.5	0.4	0.1	1.1	2.94	3.45
200	1.5	0.4	0.1	1.3	3.31	4.00
200	1.5	0.4	0.3	1.1	2.79	3.99
200	1.5	0.4	0.3	1.3	3.13	4.69
Average					3.16	3.29

The mean and variance of the load can be estimated with traditional approaches, but engineers must be careful to use best estimates rather than conservative values. The error ratio, v , should be estimated from observed failure rates and the reliability implied in traditional methods as the ratio of average nominal to average true reliability indices. It is reasonable to believe that the ratio depends on the type of limit state and the complexity of the problem, because it may be easier for engineers to detect errors in simple or common design projects. Finally, the mean and variance of the resistance, which are the only two unknowns, are a function of the random variables and the final design. The statistical properties of the geotechnical parameters can be determined from the site exploration and available studies. Therefore, the only quantity required to use the closed-form solution is the COV of the capacity, COV_Q .

Although COV_Q can be computed with MCS, the purpose of this simplified method is to avoid probabilistic calculations. Therefore, a tabular approach similar to LRFD can be used to compute COV_Q for a particular ULS equation. After the five parameters have been estimated, designers can use Equations 4-14 and 2-7 to calculate the optimum global safety factor.

6.4 Example 1: Spread Footing on Sand

The same problem presented in Chapter 5 is considered here to illustrate the simplified approach. The problem description and equations used are given in Section 5.4.

Using direct MCS, the COV of the capacity term was calculated for different values of the design parameters. During the numerical experiments, it was determined that the ratio of foundation depth to width (D/B), and the mean and COV of the soil unit weight, do not have a significant effect on the result. The most relevant parameters are

the mean and COV of the friction angle. Results in Table 6-2 assume that the friction angle and unit weight are random variables with normal distributions, the unit weight of the sand is 16 kN/m^3 , and its COV is equal to 0.10.

Table 6-2. COV_Q for shallow foundations in sand

Mean ϕ	COV_ϕ	COV_Q	
		normal ϕ	lognormal ϕ
30	0.05	0.20	0.20
	0.10	0.40	0.38
	0.15	0.64	0.57
35	0.05	0.26	0.25
	0.10	0.52	0.48
	0.15	1.00	0.77
40	0.05	0.32	0.31
	0.10	0.74	0.63
	0.15	2.20	1.07
45	0.05	0.41	0.40
	0.10	1.16	0.86
	0.15	> 6.00	1.64

Notice that COV_Q values increase rapidly with COV_ϕ . There are two possible causes for this result. The actual COV for the friction angle may be lower than the point estimates reported in the literature. When COV_ϕ is determined from tests on multiple samples from a site, the variability can be high. However, it is known that real foundations mobilize volumes of soil much larger than typical undisturbed samples, averaging local values of the resistance. Therefore, the effective COV may be lower. Another reason is that the friction angle was modeled as a normal random variable. A COV of 0.15 for $\phi = 45$ degrees implies that it is quite possible to have very high

angles of friction. One standard deviation above the mean corresponds to $\phi = 51$ degrees and two standard deviations correspond to $\phi = 59$ degrees. Other distributions with upper bounds may be more appropriate for the friction angle. Nonetheless, the results clearly show that the variability of the resistance is very sensitive to the selection of statistical parameters.

For comparison purposes, the same values used in Chapter 5 for the full optimization framework are used here. The angle of friction has a mean of 35° and $COV = 0.15$.

From Table 6-2, $COV_Q = 0.77$ for ϕ with lognormal distribution. Assuming that the load also has a lognormal distribution with $COV_F = 0.3$, $k = 0.5$, $g = 100$, and $v = 1.1$, Equations 4-14 and 2-7 give a target nominal reliability index = 2.48 and optimum safety factor = 7.6.

In comparison, a full probabilistic optimization procedure using FORM, such as the example shown in Section 5.4, yields a nominal reliability index = 2.38 and an optimum safety factor = 8.2.

The difference between the simplified procedure and a full probabilistic optimization calculation can be relatively high. For the previous example, the difference in reliability index is 4.2%, but the difference in the probability of failure is 24%. Any simplification in probabilistic methods will result in a loss of accuracy. However, the simplified method enables designers to select better target reliabilities with a simple table format similar to the procedures used in current geotechnical design. Moreover, using fixed target reliability indices can produce designs that are very far from the minimum life-cycle expected cost.

6.5 Example 2: Drilled Shaft on Clay

Another example of the simplified optimization method shows the results for the design of a drilled shaft. Similarly, cost parameters can be determined from specific project conditions. In this case, using a shaft diameter of 1.2 m (4 ft) results in an initial cost, $C_0 = \text{USD}332$, and a normalized cost ratio, $k = 1.5$.

The typical random parameters for the design of deep foundations in clays are the undrained strength, s_u , and unit weight, γ . The equations used to calculate the resistance term, Q , are shown in Chapter 5. A parametric study showed that the main variables that affect the variability of the resistance are COV_{s_u} , the mean value of s_u , and the ratio D/B . Results of MCS show the relationship between these three parameters and the COV of the resistance term, as given in Table 6-3. These MCS results assume that the soil unit weight is a random variable with mean equal to 18 kN/m^3 and COV equal to 0.10. The unit weight of concrete was assumed to be deterministic and equal to 24 kN/m^3 .

Following the values selected in the example of Chapter 5, the mean undrained strength is 100 kN/m^2 and COV_{s_u} is 0.30. For a first estimate, select $D/B = 5$, knowing that if the resulting design is significantly different, the value should be updated. According to these assumptions, COV_Q is approximately equal to 0.18.

Then, Equations 4-14 and 2-7 are used with the following parameters: $k = 1.5$, $\text{COV}_F = 0.3$, $v = 1.1$, and $g = 100$. The resulting optimum reliability index and safety factor are $\beta_{n_{\text{opt}}} = 2.84$ and $\text{FS} = 2.54$.

Table 6-3. COV_Q for drilled shafts in clay

COV of s_u	D/B	COV_Q		
		mean s_u 50 kN/m ³	mean s_u 100 kN/m ³	mean s_u 200 kN/m ³
0.15	2	0.10	0.10	0.11
	5	0.08	0.09	0.10
	10	0.07	0.08	0.09
0.30	2	0.20	0.22	0.23
	5	0.15	0.18	0.20
	10	0.14	0.17	0.19
0.45	2	0.29	0.32	0.34
	5	0.23	0.27	0.29
	10	0.20	0.25	0.28
0.60	2	0.37	0.41	0.44
	5	0.29	0.34	0.37
	10	0.26	0.32	0.36

In both examples, the optimum reliability indices and optimum safety factors were estimated using simple tables without FORM or MCS calculations. This alternative is a very important requisite for implementation in actual design methods, because full reliability optimization approaches may not be justified in simple projects.

6.6 Summary

A novel simplified approach to include consequences of failure was presented in this chapter. One fundamental way to achieve uniform reliability indices in geotechnical design is MRFD, which assigns resistance factors to each term of the capacity depending on the statistical properties of the materials. When life-cycle costs are included, additional parameters must be considered, increasing significantly the number of coefficients required for design.

An alternative method avoids optimization algorithms by using the closed-form solution presented in Chapter 4. This solution is an algebraic equation and assumes that the load and the capacity have lognormal distributions. It only requires two cost parameters, a human error factor, and the COVs of the load and capacity. To avoid probabilistic methods, the resistance COV is obtained from tables based on statistical properties of the parameters for each failure mode. The nominal reliability index, obtained from a closed-form solution, is then used to compute the optimum safety factor directly, assuming that the probability distributions of the load and resistance terms are lognormal.

Although the process to obtain the target reliability index may seem too complex, it solves a fundamental problem of the fixed target approach. In some cases, it is not possible to achieve the set target reliability, because the uncertainty in the design parameters is too large.

The two examples presented show that the simplified approach and the full reliability optimization methods give similar results. The assumption of lognormal load and resistance is reasonable for most cases, because the capacity is typically the product of several independent random variables. According to the central limit theorem in the log domain, the distribution of the product of many independent random variables is lognormal.

CHAPTER 7

SUMMARY, CONCLUSIONS AND SUGGESTED FUTURE RESEARCH

A number of alternative design approaches and results have been presented herein to consider life-cycle costs in the design of foundation structures. Different solutions to three difficult problems in design optimization are shown in Chapters 4, 5, and 6. First, the difference between nominal and true probability of failure was addressed. Second, a full reliability optimization framework using the first order reliability method (FORM) and Monte Carlo simulation (MCS) was presented. Finally, a simplified method for foundation design was proposed.

At the present time, these solutions can not be applied directly to foundation engineering design. The design optimization framework proposed in Chapter 5 requires smooth, explicit design equations and cost functions. Similarly, the simplified method proposed in Chapter 6 requires additional cost parameters and various tables to estimate statistical properties of the resistance term and optimum target reliabilities. However, these models are an important step to create new design methodologies that explicitly take into account the consequences of failure and construction costs in a rational manner.

7.1.1 Human Errors in Reliability Calculations

Two quantitative approaches were proposed in Chapter 4 to include the possibility of human errors in the design, construction, and use of geotechnical structures. The first approach assumes that a deviation from the “correct” result is a normally distributed

random variable. In the second approach, it is assumed that the ratio of the nominal over the true reliability index is constant.

The first approach is more logical because, according to one interpretation, errors are deviations from correct or nominal results. Assuming that those errors are involuntary and independent of the design parameters, their probability distributions should be symmetrical with mean equal to zero. A major obstacle to use this method is a proper way to select the coefficient of variation of the error term (COV_e). Currently, the most sensible way is to back-calculate COV_e from the difference between the true and nominal reliabilities of existing structures.

The second approach, which uses the error ratio, v , has a very important advantage: a closed-form solution can be derived to compute an optimum reliability index. But there are fewer arguments to support the assumption that a particular project will have a constant error ratio, v .

Both methodologies are quantitative and require only one additional parameter to estimate the true probability of failure. This characteristic is essential in FORM optimization, and it adds flexibility for the simplified optimization presented in Chapter 6.

7.1.2 Life-Cycle Cost Optimization Using FORM

Life-cycle cost optimization has been performed in the past using complex and inefficient methods such as MCS. A new method to determine optimum safety levels and optimum designs using FORM reduces the computational effort required by simulation techniques and eliminates a double loop optimization problem of other

formulations. Two examples for foundation design showed that both methods reach very similar solutions for typical input parameters. Furthermore, optimization using FORM does not suffer some of the problems of MCS, such as negative values of capacity or load in some simulations. In addition, the method can be used with non-normal distributions or correlated parameters. And all of these calculations can be performed in a spreadsheet with nonlinear optimization capabilities.

Probably, the only major drawback of FORM optimization is that the inverse calculation of the most probable failure point (MPP) introduces an error. The reliability index obtained with the inverse approach can be different from the one calculated with direct FORM. This error occurs because the search direction of the method is given by the derivative of the performance function evaluated at the mean values of the design parameters. Therefore, each performance function must be studied to determine if the error is acceptable.

It is important to notice that the optimum reliability index is only an intermediate result in FORM optimization. In reality, the most important result is the optimum set of values of all the design variables. The second example presented in Chapter 5 considers a design variable that has only discrete values (shaft diameter, B). In that case, it is possible to run separate optimization calculations for each case and later compare their total costs.

A sensitivity analysis for each example in Chapter 5 showed that the normalized cost of failure is the most sensitive parameter for design optimization. Therefore, adequate selection of failure costs is a fundamental task to achieve an optimum design. It will be necessary to issue guidelines in this topic. For larger projects, geotechnical and

structural engineers should work together with experts in economic valuation to select appropriate cost parameters.

7.1.3 Simplified Optimization

The proposed simplified cost optimization framework employs a methodology similar to the one used by load and resistance factor design (LRFD) codes. It is based on tables and algebraic equations, but it includes the relative cost of failure and the normalized cost ratio. In a typical design process, first the designer must define the variability parameters of some random variables and select from a table the corresponding coefficient of variation (COV) of the resistance. Then, he or she must estimate the relative cost of failure, g , and the normalized cost ratio, k , based on the cost of two or more design options with different reliability. Finally, the optimum reliability index, β_{opt} , and the optimum safety factor, FS_{opt} , are calculated using two algebraic equations.

One drawback of the simplification is that only discrete values of some design parameters can be used. For example, tables only include some values for COVs or mean values, and the designer must interpolate between the closer values included in the list. However, linear interpolation is a simple task. Moreover, this numerical procedure is no more complex than current LRFD approaches.

7.2 Conclusions

The mean value first order second moment (FOSM) method is not accurate enough for reliability-based design optimization problems and should not be used. Even when the

safety margin is used as the performance function, most resistance equations include nonlinear combinations of the design parameters that can not be addressed correctly by FOSM.

The new method using FORM is a notable improvement over previous optimization methods because it:

- requires fewer assumptions
- can be implemented using widely available software
- finds optimum values of design variables

On the other hand, designers must not only estimate direct and indirect costs, but they have to define smooth cost functions before the optimization is performed. Some engineers may find it difficult to estimate failure costs with confidence, because it is a task not required in traditional design methods.

The proposed optimization procedure using FORM agrees well with previous simplified closed-form solutions and with MCS optimization results.

All the optimization methods discussed can be modified to include human errors as random variables. The mean of the error term should be equal to zero, and the coefficient of variation must be computed from observed failure rates. Since the nominal reliability is approximately one order of magnitude greater than the true value, the resulting COV of the error must lie between 0.3 and 0.5 for typical foundation systems.

For the simplified method, no cost functions are required, but two additional parameters are needed. The initial cost coefficient, k , and the normalized cost of failure, g , can vary significantly depending on the type of project and type of

foundation. Previous studies found that the values of k were around 0.05 to 0.1 for common structures (Kanda and Ellingwood 1991). However, the examples presented herein showed that k can be much higher for foundations (0.5 to 1.5). Fortunately, the coefficient, k , can be calculated according to the specific cost conditions of each project.

The proposed simplified optimization can be useful in several scenarios, for example:

- when only approximate results are needed
- for validating full optimization results
- for including optimization methods in building codes

7.3 Suggested Future Research

It may be argued that the methods presented herein should not be used in practice, because there are some issues that require further study. It is true that some assumptions do not reflect real conditions in building projects. However, by definition, all models are simplifications of real systems. Furthermore, these methods represent improvements over current design approaches.

Future work in reliability optimization should include:

- effect of economic optimization in total construction costs
- effect of uncertainty in initial costs
- reliability of multi-component systems, redundancy, and ductility
- selection of human error parameters
- estimation of failure costs
- implementation in building codes

It is unknown what will be the impact of using economic optimization methods in design practice. Currently, cost optimization is essentially empirical. If the proposed methods are included in building codes and used in practice, savings in a single project may not be noticeable because the probability of failure is very small. A study to quantify the impact of cost optimization should evaluate a representative sample of different construction projects and compare real costs with expected costs using life-cycle costs optimization (LCCO).

Another issue that requires further study is the effect of the uncertainty in initial costs. Obviously, foundations must be designed before they are built. Therefore, the final construction cost is not known with certainty until the work is complete. If construction costs are needed for the design of structures, only approximations can be used. However, the cost of some structures is less variable than others. Typically, the construction cost of foundations is highly variable, because unexpected ground conditions can alter the design or the schedule.

Another related problem is the evaluation of multi-component systems, redundancy, and ductility. Assessment of the reliability of multi-component systems is challenging, because the relationship between components can vary. For example, a foundation system composed of spread footings connected with beams has some redundancy. However, the redundancy level depends on the final design. Therefore, an explicit relationship between system reliability and design variables for each foundation type is necessary.

Assessment of human error parameters is not easy and may be controversial, because each project is different. The magnitude of errors may vary depending on the location, building code used, and type of project. However, it is possible to select a sample of

representative structures in a region and compare nominal and true reliability levels. This exercise should help to select better values for the variability of designs caused by human errors.

Optimum designs are very sensitive to failure costs, according to the objective functions considered in Chapter 5. However, these costs are rarely considered during routine foundation design. Furthermore, failure costs also are random variables, although they are assumed to be deterministic. In the author's opinion, it is necessary to develop a simplified framework for engineers to select potential failure costs based on the type of project. It could be useful to define bounds for failure costs in typical projects and calculate optimum designs for those bounds. This exercise will give engineers a sense for the importance of failure cost in the final design.

Finally, it is necessary to study the convenience of implementing simplified optimization in building codes. It is feasible that a method similar to the one proposed in Chapter 6 could be implemented in foundation design codes. Good judgment and experience always will be necessary for efficient foundation design, but optimization methods can improve designs rationally and reduce total expected costs.

Full reliability methods in geotechnical engineering have not found their way into common design practice yet. One of the reasons for this lag is that most geotechnical engineering courses in undergraduate and graduate programs do not cover these topics. Reliability-based design (RBD) is covered infrequently in graduate programs, while optimization courses are rarely offered. Fortunately, RBD optimization (RBDO) methods can be applied to any engineering design problem with explicit performance functions and cost equations. It is not necessary to offer special courses for civil engineering students. RBDO is a very active and growing research field with

contributions from many engineering fields with similar design problems. Hopefully this work will bring attention to the problem and serve as a basis for new methods on applied optimization.

APPENDIX A

CALIBRATION OF LRFD EQUATIONS

The load and resistance factors used in simplified LRFD methods can be adjusted to achieve a specific target reliability. Consider the safety margin equation ($M = Q - F$). The deterministic form can be transformed to a probabilistic problem considering the load and resistance as random variables with known distributions. If the load and resistance are normally distributed, the reliability index is calculated with Equation 2-6, shown again here:

$$\beta = \frac{m_Q - m_F}{\sqrt{s_Q^2 + s_F^2}} \quad (\text{A1-1})$$

in which m_Q and m_F = mean of resistance and load terms and s_Q and s_F = standard deviation of resistance and load terms, respectively.

The denominator can be linearized as (Scott et al. 2003):

$$\sqrt{s_Q^2 + s_F^2} \approx \alpha(s_Q + s_F) \quad (\text{A1-2})$$

in which α = separation coefficient.

The separation coefficient is not a constant. If only one load and one resistance variable are considered, α can vary from 0.707 to 1.0. According to Scott et al. (2003), the COV of the resistance typically varies from 0.1 to 0.5, and the COV of the load varies from 0.1 to 0.25. For these COVs, the resulting separation coefficient varies between 0.7 and 0.85, but for practical purposes it is assumed equal to 0.75. This

approximation is relatively good for most applications. If the two standard deviations are identical, the error of the approximation is 6%. In the worst case, if the standard deviations are very different numerically, the error is 25%, because when one standard deviation is much larger than the other, the real separation coefficient is equal to one. For other cases, the result is closer to the real value. When the ratio of the major to the minor standard deviation (e.g. s_Q/s_F) is two, the error is 0.6%. Figure A1-1 shows the approximation error as a function of the standard deviation ratio.

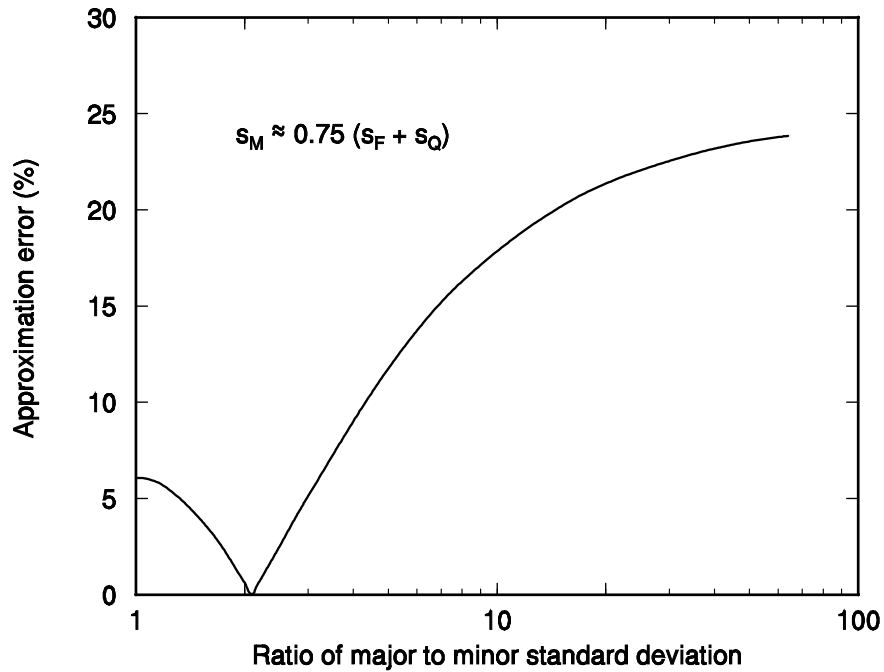


Figure A1-1. Error of the linear function to approximate the standard deviation of the safety margin

Then, the reliability index is approximated as:

$$\beta \approx \frac{m_Q - m_F}{0.75 (s_Q + s_F)} \quad (\text{A1-3})$$

Equation A1-3 can be rearranged as follows:

$$(1 - 0.75 \beta_T \text{COV}_Q) m_Q = (1 + 0.75 \beta_T \text{COV}_F) m_F \quad (\text{A1-4})$$

By comparing Equations A1-4 and 2-18, the load and resistance factors are given by:

$$\psi = (1 - 0.75 \beta_T \text{COV}_Q) m_Q / Q_N \quad (\text{A1-5})$$

$$\eta = (1 + 0.75 \beta_T \text{COV}_F) m_F / F_N \quad (\text{A1-6})$$

In this simplified approach, the resistance factor is not a function of the load factor, but they are related. Therefore, the resistance factor obtained with Equation A1-5 must be used in combination with the load factor from Equation A1-6 to achieve the desired target reliability.

Unlike the first case, the calibration equation for lognormal distributions contains the load factor. In principle, there are an infinite number of factor combinations that can achieve a given reliability. For practical purposes, a load factor greater than one must be selected depending on the variability of the load. An optimal load factor is the value that achieves consistent reliabilities for different coefficients of variation of the resistance.

When the load and resistance have lognormal distributions, the reliability index is defined by Equation 2-7. If the nominal resistance is equal to the mean value (no bias), then the mean resistance is:

$$m_Q = Q_{\text{factored}} / \psi \quad (\text{A1-7})$$

Substituting Equation A1-6 into Equation 2-7 and solving for ψ gives the following:

$$\psi = \frac{Q_{\text{factored}} \left[(1 + \text{COV}_F^2) / (1 + \text{COV}_Q^2) \right]^{0.5}}{m_F \exp \left\{ \beta_T \left(\ln \left[(1 + \text{COV}_F^2)(1 + \text{COV}_Q^2) \right] \right)^{0.5} \right\}} \quad (\text{A1-8})$$

Since the factored resistance, Q_{factored} , must be equal to the factored load, m_F , they cancel out in Equation A1-8, and the result is Equation 2-22.

APPENDIX B

USE OF MICROSOFT EXCEL SOLVER

As other authors have noted (Low and Tang 1997, Phoon 2008), optimization using the GRG nonlinear engine in Excel must be done carefully to avoid incorrect or inaccurate results. The optimization procedure presented in Chapter 5 can be done in two ways: using the dialog box that appears when Solver is called or using a macro. In general, it is easier to use the dialog box. However, using a macro to run Solver is faster and can be useful when several scenarios are evaluated.

Solver Using the Dialog Box

For the first method, the user must open the Solver Dialog Box by clicking on the menu *Data - Solver*. In the dialog box, the first input field is called Cell Objective. This field defines the cell that contains the expected cost equation, $E(C)$. In the second field, there are three option buttons called “Max”, “Min”, and “Value of”. The option called “Min” must be selected, because the minimum expected cost is sought. The field called “By Changing Variable Cells” must specify the range of cells that contain design parameters, such as foundation depth or width, and the cell with the reliability index (Figure A2-1).

If all the design variables must be positive, such as dimensions, it is advisable to select the option labeled “Make Unconstrained Variables Non-Negative” to prevent searches in unfeasible regions. Also “GRG Nonlinear” must be selected from the three options in the combo box.

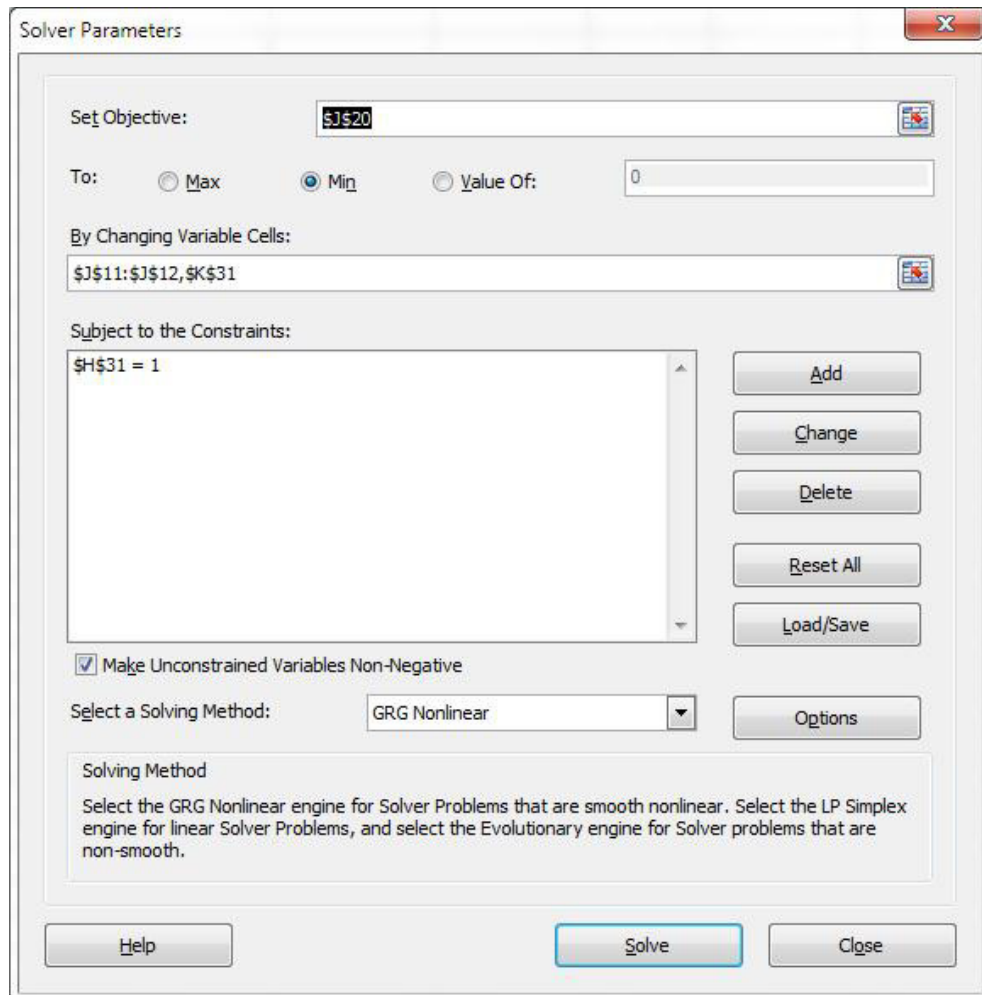


Figure A2-1. Dialog box for Excel Solver

Next, select the button called “Options” to open the dialog box shown in Figure A2-2. The value of an option called “Constraint Precision” can affect the result in Excel Solver. By default, this value is set to 10^{-6} , but it can lead to variations in the results in consecutive runs of the same problem. Results presented herein were obtained using a constraint precision of 10^{-13} . It may seem excessive to use such high precision. However, calculations in Excel have a precision of 15 digits, and the increase in calculation time is not noticeable. Default values can be used for the rest of the options in this dialog box.

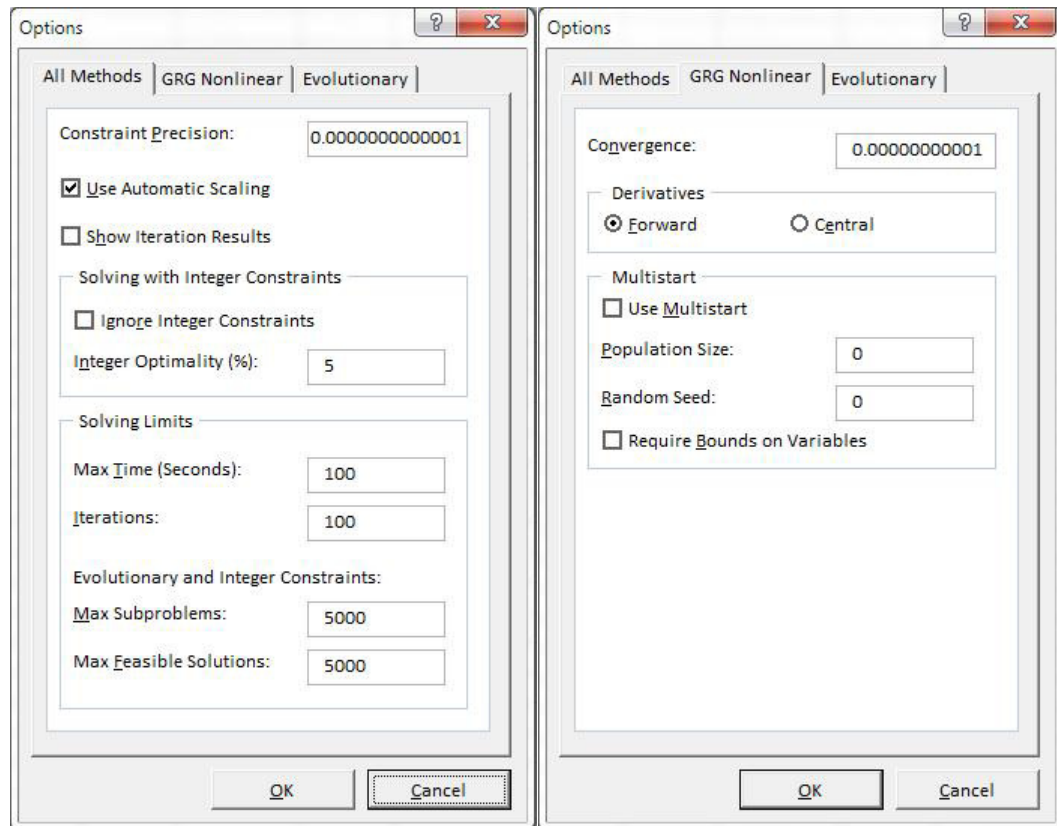


Figure A2-2. Option menus for Excel Solver

The next dialog box appears after selecting the tab named “GRG Nonlinear”. Convergence is also an important parameter that determines if a solution has been found. Examples presented herein were calculated using a convergence of 10^{-11} . This value may not be critical in FORM optimization but can lead to inconsistencies in MCS optimization.

Solver Using Macros

The same problems can be solved using Visual Basic for Applications (VBA) saved as a macro in Excel. Figure A2-3 shows an example of the code used for FORM optimization. All the options available in the dialog box method can be defined in VBA.

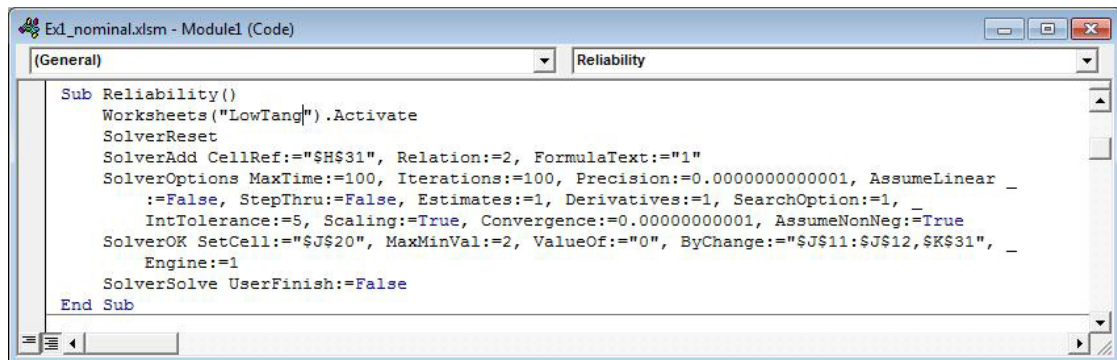


Figure A2-3. Macro to call Excel Solver

The spreadsheet used with the code of the VBA is shown in Figure A2-4.

	A	B	C	D	E	F	G	H	I	J
2										
3		Reliability-Based Foundation Design Optimization								
4										
5		Instructions:								
6		Edit the blue cells with the appropriate values. Click the button "Optimize" to run the optimization code.								
7										
8										
9		Soil properties					Optimum Design			
10		Name	Units	Mean	COV		Name	Value		
11		Angle of friction	deg	35.0	0.15		Depth, D	2.483484		
12		Unit weight	kN/m3	16.0	0.1		Width, B	3.0999		
13		Max axial load	kN	3,000	0.3		Thickness, H	0.464985		
14							L/B	1		
15							Pedestal, S	0.3		
16		Cost					Overexc, BO	0.620871		
17		Name	Units	Unit Cost						
18		Concrete	m3	\$200.00			Initial Cost	\$2,699.94		
19		Excavation	m3	\$25.00			Exp. fail cost	\$465.60		
20		Fill	m3	\$5.00			Expected cost	\$3,165.54		
21		Formwork	m2	\$50.00						
22										
23		Covariance matrix					Design Point			
24										
25		27.5625	0	0			ϕ	γ	load	
26		0	2.56	0			19.019	15.343	4108.874268	
27		0	0	810000						

Figure A2-4. Spreadsheet for foundation design optimization with FORM.

APPENDIX C

RANDOM VARIABLES IN EXCEL

Excel has a number of built-in functions that can generate pseudorandom numbers with different distributions. These values are considered pseudorandom because they come from algorithms as opposed to a real random process. Strictly speaking, variables generated by any conventional computer engine must be called pseudorandom. However, for simplicity they are called random in this document. Broader discussions on random number generation can be found elsewhere (Baecher and Christian 2003).

To generate random variables, it is necessary to use the inverse distribution function. If the inverse distribution function is available, then, a realization of the random variable is calculated as:

$$x = F^{-1}[n] \quad (A3-1)$$

in which $F^{-1}[\cdot]$ = inverse probability function and n = standard uniform random variable.

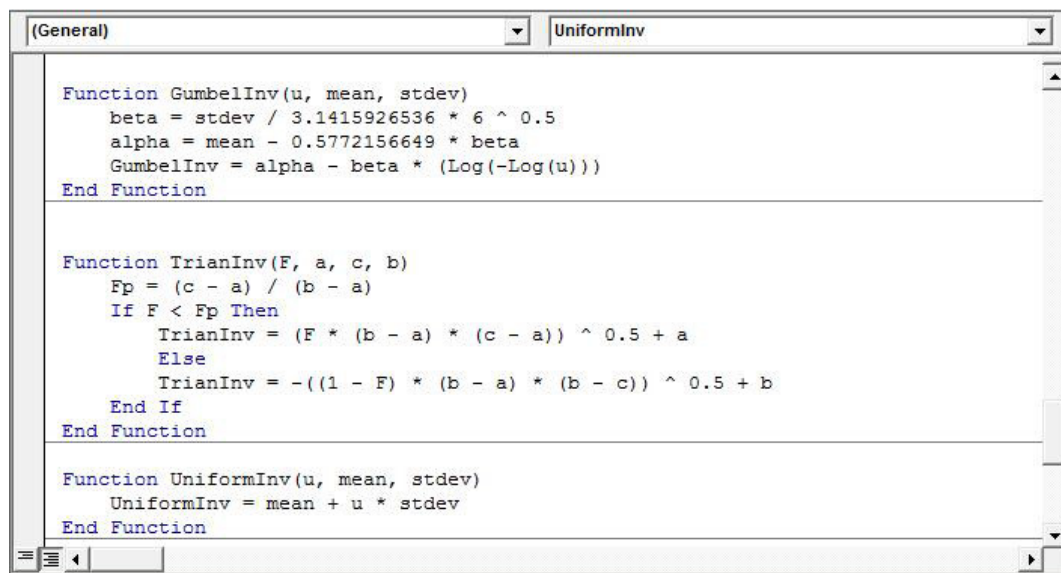
Table A3-1 shows a list of available inverse distributions in Excel 2010. If the user wants to generate random variables with other distributions, other methods must be used.

Table A3-1. Inverse probability functions in Excel

Distribution	Function
Normal	=NORM.INV(probability, mean, standard_dev)
Lognormal	=LOGNORM.INV(probability, mean, standard_dev)
Beta	=BETA.INV(probability, alpha, beta, [A], [B])
Gamma	=GAMMA.INV(probability, alpha, beta)
Chi-squared	=CHISQ.INV(probability, deg_freedom)
F	=F.INV(probability, deg_freedom1, deg_freedom2)

Uncorrelated Variables

A practical approach to generate uncorrelated random variables not included in Excel (e.g. Gumbel, triangular, uniform, etc.) is to write the inverse functions in VBA and then use those functions normally in a spreadsheet (Figure A3-1). Other probability distributions are presented in Low and Tang (2007).



```

Function GumbelInv(u, mean, stdev)
    beta = stdev / 3.1415926536 * 6 ^ 0.5
    alpha = mean - 0.5772156649 * beta
    GumbelInv = alpha - beta * (Log(-Log(u)))
End Function

Function TrianInv(F, a, c, b)
    Fp = (c - a) / (b - a)
    If F < Fp Then
        TrianInv = (F * (b - a) * (c - a)) ^ 0.5 + a
    Else
        TrianInv = -((1 - F) * (b - a) * (b - c)) ^ 0.5 + b
    End If
End Function

Function UniformInv(u, mean, stdev)
    UniformInv = mean + u * stdev
End Function

```

Figure A3-1. Function definitions in VBA for Excel

Correlated Variables

In general, a multivariate density function can be defined by a vector and a covariance matrix (Eq. A3-2)

$$\mu = \begin{Bmatrix} \mu_1 \\ \mu_1 \\ \vdots \\ \mu_n \end{Bmatrix} \quad (A3-2)$$

$$C = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1n}\sigma_1\sigma_n \\ \rho_{21}\sigma_2\sigma_1 & \sigma_2^2 & & \rho_{2n}\sigma_2\sigma_n \\ \vdots & & \ddots & \vdots \\ \rho_{n1}\sigma_n\sigma_1 & \rho_{n2}\sigma_n\sigma_2 & \cdots & \sigma_n^2 \end{bmatrix}$$

in which μ_i = mean of the i^{th} random variable, σ_i = standard deviation the i^{th} random variable, and ρ_{ij} = Pearson's correlation coefficient

The most common way to generate random correlated variables in Excel is to transform vectors of independent normally distributed random values into vectors of correlated random values. In summary, the transformation consists of the following steps:

1. Obtain the lower triangular matrix L using Choleski decomposition of the covariance matrix C . Note that $C = LL^T$ but C must be positive definite.
2. Multiply the vector of independent random variables times the matrix L^T to obtain the vectors of correlated random values.

APPENDIX D

FORM OPTIMIZATION WITH NON-NORMAL, CORRELATED VARIABLES

As noted in Chapter 5, the most probable failure point (MPP) depends on the correlation structure of the random variables. In the proposed optimization framework, this correlation structure can be included in a spreadsheet (Figure A4-1) using the procedure described by Low and Tang (2007).

The process includes the following steps:

1. Define input parameters (mean, variance, correlation matrix, distributions, unit costs), performance function, and objective function.
2. Set initial values for the design point, x' (mean values can be used) and the reliability index, β' (typically between 2 and 3).
3. Calculate equivalent normal means and variances for non-normal distributions with Equations A4-1 and A4-2.

$$\mu^N = x' - \sigma^N \Phi^{-1} [F(x')] \quad (A4-1)$$

$$\sigma^N = \frac{\phi\{\Phi^{-1} [F(x')]\}}{f(x')} \quad (A4-2)$$

in which $F(\cdot)$ = non-normal cumulative probability function and $f(\cdot)$ = probability density function.

The VBA code to transform several distributions was presented by Low and Tang (2007) and was used herein.

4. Obtain the eigenvectors and eigenvalues of the correlation matrix.
5. Calculate the updated design point, x^* , using Equation 5-8.
6. Calculate the reliability index in a cell using the following function:
$$\sqrt{(x^* - \mu^N)^T C^{-1} (x^* - \mu^N)} .$$
7. Run Solver with the following options (Figure A4-2):
 - Minimize: total expected cost, $E(C_T)$.
 - By changing: design parameters, d , initial reliability index, β' , and the initial values of the design point, x' .
 - Subject to: $F(x^*) = 1$ and $x' = x^*$.
8. Run Solver again with different initial values of x' and β' to confirm that the solution is stable.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Reliability-Based Foundation Design Optimization											
2													
3		Instructions:											
4		Edit the blue cells with the appropriate values. Run Solver to find the optimum parameters											
5													
6		Random Parameters											
7		Name	Units	Distribution	Param 1	Param 2	Param 3	x'	μ^N	σ^N	dg/dxi	$(s*dg/dxi)^2$	x^*
8		Angle of friction	deg	Lognormal	35.0	5.25		24.51	33.0	3.656379	0.91204	11.1206	24.51
9		Unit weight	kN/m3	Normal	16.0	1.6		13.91	16.0	1.6	0.76282	1.48966	13.91
10		Max axial load	kN	Lognormal	3,000	900		5,916	1643.5	1736.829	-0.00223	4.02385	5,916
11												4.078499	
12		Correlation matrix											
13		1	0.25	0									
14		0.25	1	0									
15		0	0	1									
16		Covariance matrix											
17		13.3691081	1.46255164	0									
18		1.46255164	2.56	0									
19		0	0	3016576.6									
20		Eigenvectors											
21		0.70710678	-0.70710678	0									
22		0.70710678	0.70710678	0									
23		0	0	1									
24		Eigenvalues											
25		1.25	0	0									
26		0	0.75	0									
27		0	0	1									
28		Cost											
29		Name	Units	Unit Cost									
30		Concrete	m3	\$200.00									
		Excavation	m3	\$25.00									
		Fill	m3	\$5.00									
		Formwork	m2	\$50.00									
		Optimum Design											
		Name	Units	Value									
		Depth	m	2.2898									
		Width	m	2.9258									
		Thickness	m	0.439									
		L/B	-	1									
		Pedestal, S	m	0.3									
		Overexc, B0	m	0.57245									
		Initial Cost		\$2,254.45									
		Total expected cost		\$2,524.04									
		β'	F	β	Pf	Cf	E(F)	SD(F)					
		3.459	1	3.459	0.000271	\$996,000	6.687	1.644					

Figure A4-1. FORM optimization with non-normal, correlated random variables

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

-
-
-
-

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Buttons: Add, Change, Delete, Reset All, Load/Save, Options, Help, Solve, Close

Figure A4-2. Solver options for FORM optimization

REFERENCES

- American Association of State Highway and Transportation Officials (AASHTO) (2004). *LRFD Bridge Design Specifications, 3rd Ed.*, AASHTO,. Washington, DC.
- American Society of Civil Engineers (ASCE) (2005). "Minimum design loads for buildings and other structures." *SEI/ASCE 7-05*, ASCE, Reston, VA.
- Abdelatif Hassanien, S. S. (2007). "Optimization under uncertainty for structural systems." Ph.D. thesis, University of Calgary, Calgary, Alberta.
- Allen, T. M. (2005). "Development of geotechnical resistance factors and downdrag resistance factors for LRFD foundation strength limit design: reference manual". *Report FHWA-NHI-05-052*, Federal Highway Administration, Washington, DC.
- Australia-New Zealand Committee on Large Dams (ANCOLD) (1994). *Guidelines on Risk Assessment 1994*, ANCOLD, Hobart, Australia.
- Ang, A. H.-S. and De Leon, D. (1997). "Determination of optimal target reliabilities for design and upgrading of structures." *Structural Safety*, 19(1), 91-103.
- Aoues, Y. and Chateauneuf, A. (2008). "Reliability-based optimization of structural systems by adaptive target safety - Application to RC frames." *Structural Safety*, 30(2), 144-161.
- Babu, G. L. S. and Basha, B. M. (2008). "Optimum design of cantilever retaining walls using target reliability approach." *International Journal of Geomechanics*, 8(4), 240-252.

- Baecher, G. B. (1987). "Geotechnical risk analysis user's guide". *FHWA/RD-87-011*, Federal Highway Administration, McLean, VA.
- Baecher, G. B. and Christian, J. T. (2003). *Reliability and statistics in geotechnical engineering*. John Wiley & Sons, Chichester, England.
- Barker, R. M., and Puckett, J. A. (2007). *Design of highway bridges: an LRFD approach*. John Wiley & Sons, Hoboken, NJ.
- Basha, B. M. and Babu, G. L. S. (2008). "Target reliability based design optimization of anchored cantilever sheetpile walls." *Canadian Geotechnical Journal*, 45(4), 535-548.
- Bea, R. (2000). "Performance shaping factors in reliability analysis of design of offshore structures." *Journal of Offshore Mechanics and Arctic Engineering*, 122(8), 163-172.
- Bea, R. (2006). "Reliability and human factors in geotechnical engineering." *Journal of Geotechnical and Geoenvironmental Engineering*, 132(5), 631-643.
- Blockley, D. (1999). "Risk based structural safety methods in context." *Structural Safety*, 21(4), 335-348.
- Brown, C. B. and Yin, X. (1988). "Errors in structural engineering." *Journal of Structural Engineering*, 114(11), 2575-2593.
- Brown, C. B., Elms, D. G. and Melchers, R. E. (2008). "Assessing and achieving structural safety." *Proceedings, Institution of Civil Engineers: Structures and Buildings*, 161(4), 219-230.

- Building Officials and Code Administrators International (BOCA) (1993). *The BOCA national building code*. BOCA, Country Club Hills, IL.
- Castillo, E., Minguez, R., Ruiz-Teran, A. and Fernandez-Canteli, A. (2004). "Design and sensitivity analysis using the probability-safety-factor method. An application to retaining walls." *Structural Safety*, 26(2), 159-169.
- CEN (2001). "Eurocode 7 Part 1: geotechnical design: general rules". *EN 1997-1*, European Committee for Standardization (CEN), Brussels, Belgium.
- Chen, Y. J. and Kulhawy, F. H. (1994). "Case history evaluation of the behavior of drilled shafts under axial and lateral loading". *Report TR-104601*, Electric Power Research Institute, Palo Alto, CA.
- Christian, J. T. and Baecher, G. B. (1999). "Point-estimate method as numerical quadrature." *Journal of Geotechnical and Geoenvironmental Engineering*, 125(9), 779-786.
- Christian, J. T. and Baecher, G. B. (2001). "Discussion of Factors of safety and reliability in geotechnical engineering." *Journal of Geotechnical and Geoenvironmental Engineering*, 127(8), 700-703.
- Cui, L. and Sheng, D. (2005). "Genetic algorithms in probabilistic finite element analysis of geotechnical problems." *Computers and Geotechnics*, 32(8), 555-563.
- Dasgupta, P. and Maler, K.-G. (1994). "The environment and emerging development issues." *Cost-benefit analysis*. R. Layard and S. Glaister, Eds., Cambridge University Press, Cambridge, UK., 319-348.

- Ditlevsen, O. (1979). "Narrow reliability bounds for structural optimization." *Journal of Structural Mechanics*, 7(4), 453-472.
- Duncan, J. M. (2000). "Factors of safety and reliability in geotechnical engineering." *Journal of Geotechnical and Geoenvironmental Engineering*, 26(4), 307-316.
- Ellingwood, B. R. (2001). "Acceptable risk bases for design of structures." *Progress in Structural Engineering and Materials*, 3(2), 170-179.
- Ellingwood, B. R. (1987). "Design and construction error effects on structural reliability." *Journal of Structural Engineering*, 113(2), 409-422.
- Ellingwood, B., Galambos, T. V., MacGregor, J. G. and Cornell C. A. (1980). "Development of a probability-based load criterion for American National Standard A58: Building code requirements for minimum design loads in buildings and other structures". *Special Publication 577*, U.S. Department of Commerce, National Bureau of Standards, Washington, DC.
- Environmental Protection Agency (EPA) (2009). *Inventory of U.S. greenhouse gas emissions and sinks 1991 – 2007*, EPA, Washington, DC.
- Feng, Y. S. and Moses, F. (1986). "A method of structural optimization based on structural system reliability." *Journal of Structural Mechanics*, 14(4), 437-453.
- Flyvbjerg, B., Holm, M. S. and Buhl, S. (2002). "Underestimating costs in public works projects: error or lie?" *Journal of the American Planning Association*, 68(3), 279-295.
- Frangopol, D. M. and Corotis, R. B. (1996). "Reliability-based structural system optimization: state-of-the-art versus state-of-the-practice." *Analysis and*

Computation. Proceedings, Twelfth Conference in conjunction with Structures Congress XIV, ASCE, Chicago, IL, 67-78.

Frangopol, D. M. and Maute, K. (2003). "Life-cycle reliability-based optimization of civil and aerospace structures." *Computers & Structures*, 81(7), 397-410.

Frangopol, D. M., Lin, K.-Y. and Estes, A. C. (1997). "Life-cycle cost design of deteriorating structures." *Journal of Structural Engineering*, 123(10), 1390-1401.

Fylstra, D., Lasdon, L., Watson, J. and Waren, A. (1998). "Design and use of the Microsoft Excel Solver." *Interfaces*, 28(5), 29-55.

García-Pérez, J., Castellanos, F. and Díaz, O. (2005). "Occupancy importance factor in earthquake engineering." *Engineering Structures*, 27(11), 1625-1632.

Gasser, M. and Schuëller, G. I. (1997). "Reliability-based optimization of structural systems." *Mathematical Methods of Operations Research*, 46(3) 287-307.

Gavin, K. and Xue, J. (2009). "Use of a genetic algorithm to perform reliability analysis of unsaturated soil slopes." *Geotechnique*, 59(6), 545-549.

Goh, A. T. C. (1999). "Genetic algorithm search for critical slip surface in multiple-wedge stability analysis." *Canadian Geotechnical Journal*, 36(2), 382-391.

Grigoriu, M. D. and Turkstra, C.J. (1979). "Safety of structural systems with correlated resistances." *Applied Mathematical Modelling*, 3(2), 130-136.

Harr, M. E. (1987). *Reliability-based design in civil engineering*, McGraw Hill, New York, NY.

- Hasofer, A. M. and Lind, N. C. (1974). "An exact and invariant first-order reliability format." *Journal of the Engineering Mechanics Division, ASCE*, 100(1), 111-121.
- Hendricks, V. F., Pedersen, S. A. and Jorgensen, K. F. (2001). *Probability theory: philosophy, recent history and relations to science*. Springer, New York, NY.
- Hong, H. P. (2000). "Distribution of structural collapses and optimum reliability for infrequent environmental loads." *Structural Safety*, 22(4), 297-311.
- Hong Kong Government Planning Department (HKGPD) (2005). "Urban design guidelines." Chap. 11 in *Hong Kong Planning Standards and Guidelines*. HKGPD, Hong Kong, China.
- Honjo, Y. (2008). "Monte Carlo simulation in reliability analysis." Chap. 4 in *Reliability-based design in geotechnical engineering*. Phoon K. K., Ed., Taylor & Francis, Abingdon, UK. 169-191.
- Hsu, Y.-C., Lin, J.-S. and Kuo J.-T. (2007). "Projection method for validating reliability analysis of soil slopes." *Journal of Geotechnical and Geoenvironmental Engineering*, 133(6), 753-756.
- Hynes, M. and Vanmarke, E. (1976). "Reliability of embankment performance predictions." *Proceedings, ASCE Engineering Mechanics Division Specialty Conference*, Waterloo University Press, Waterloo, Ontario, 367-383.
- International Code Council (ICC) (2000). *International building code*. ICC, Falls Church, VA.

- International Congress of Building Officials (ICBO) (1997). *Uniform building code*. ICBO, Whittier, CA.
- Jongejan, R. B., Jonkman, S. N. and Vrijling, J. K. (2005). "Methods for economic valuation of loss of life." *Proceedings: Law Economics and Management of Large-Scale Risks*, CIREQ, Montreal, Quebec.
- Junnla, S., Horvath, A., and Guggemos, A. (2006). "Life-cycle assessment of office buildings in Europe and the United States." *Journal of Infrastructure Systems* 12(1), 10-17.
- Kanda, J. and Ellingwood, B. (1991). "Formulation of load factors based on optimum reliability." *Structural Safety*, 9(3), 197-210.
- Kanda, J. and Shah, H. (1997). "Engineering role in failure cost evaluation for buildings." *Structural Safety*, 19(1), 79-90.
- Kanda, J., Takada, T. and Hang, C. (2007). "Target structural reliability in life cycle consideration." *International Journal of Risk Assessment and Management*, 7(6/7), 846-861.
- Kong, J. and Frangopol, D. M. (2004). "Cost-reliability interaction in life-cycle cost optimization of deteriorating structures." *Journal of Structural Engineering*, 130(11), 1704-1712.
- Kulhawy, F. H. and Phoon, K.-P. (1996). "Engineering judgment in the evolution from deterministic to reliability-based foundation design." *Uncertainty in the Geologic Environment (GSP 58)*, Shackelford, C. D., Nelson, P. P., and Roth, M. J. S., Eds., ASCE, New York, NY, 29-48.

- Layard, R. and Glaister, S. (1994). *Cost-benefit analysis, 2nd Ed.* Cambridge University Press, Cambridge, UK.
- Li, K. S. (1992). "Point-estimate method for calculating statistical moments." *Journal of Engineering Mechanics*, 118(7), 1506-1511.
- Liang, J., Mourelatos, Z. P. and Nikolaidis, E. (2007). "A single-loop approach for system reliability-based design optimization." *Journal of Mechanical Design*, 129(12), 1215-1225.
- Lind, N. C. (1994). "Target reliabilities from social indicators." *Proceedings, International Conference on Structural Safety & Reliability 93*. Innsbruck, Austria, 1897-1904.
- Low, B. K. and Tang, W. H. (1997). "Efficient reliability evaluation using a spreadsheet." *Journal of Engineering Mechanics*, 123(7), 749-752.
- Low, B. K. and Tang, W. H. (2007). "Efficient spreadsheet algorithm for first-order reliability method." *Journal of Engineering Mechanics*, 133(12), 1378-1387.
- Lumb, P. (1970). "Safety factors and the probability distributions of strength." *Canadian Geotechnical Journal*, 7(3), 225-242.
- Marler, R. T. and Arora, J. S. (2004). "Survey of multi-objective optimization methods for engineering." *Structural and Multidisciplinary Optimization*, 26(6), 369-395.
- Melchers, R. E. (1987). *Structural reliability: analysis and prediction*. John Wiley & Sons, Chichester, England.

- Melchers, R. E. (1989). "Human error in structural design tasks." *Journal of Structural Engineering*, 115(7), 1795-1807.
- Mendell, N. R. and Elston, R. C. (1974). "Multifactorial qualitative traits: genetic analysis and prediction of recurrence risks." *Biometrics*, 30(1), 41-57.
- Meyerhof, G. G. (1984). "Safety factors and limit state analysis in geotechnical engineering." *Canadian Geotechnical Journal*, 21(1), 1-7.
- Meyerhof, G. G. (1995). "Development of geotechnical limit state design." *Canadian Geotechnical Journal*, 32(1), 128-136.
- Moses, F. (2001). "Calibration of load factors for LRFD bridge evaluation". *NCHRP Report 454*, Transportation Research Board, Washington, DC.
- Nakanishi, S. and Nakayasu, H. (2002). "Reliability design of structural system with cost effectiveness during life cycle." *Computers & Industrial Engineering*, 42(2-4), 447-456.
- National Research Council. (2006). *Geological and geotechnical engineering in the new millennium : opportunities for research and technological innovation*. National Academy Press, Washington, DC.
- National Research Council of Canada (NRC) (1995). *National building code of Canada*. NRC, Ottawa, Ontario.
- Naval Facilities Engineering Command (NAVFAC) (1986a). *Soil mechanics. Design Manual 7.01*, NAVFAC, Alexandria, VA.

- NAVFAC (1986b). *Foundations and earth structures. Design Manual 7.02*, NAVFAC, Alexandria, VA.
- Nikolaidis, E., Chen, S., Cudney, H., Haftka, R. and Rosca, R. (2004). "Comparison of probability and possibility for design against catastrophic failure under uncertainty." *Journal of Mechanical Design*, 126(3), 386-394.
- Nowak, A. S. (1995). "Calibration of LRFD bridge code." *Journal of Structural Engineering*, 121(8), 1245-1251.
- Orr, T. L. L. and Breysse, D. (2008). "Eurocode 7 and reliability-based design." Chap. 8 in *Reliability-based design in geotechnical engineering*, Phoon, K.K., Ed., Taylor & Francis, Abingdon, UK. 298-343.
- Paikowsky, S. (2004). "Load and resistance factor design (LRFD) for deep foundations". *NCHRP Report 507*, Transportation Research Board, Washington, DC.
- Peck, R. B., Hanson, W. E. and Thornburn, T. H. (1974). *Foundation Engineering, 2nd Ed.*, John Wiley & Sons, New York, NY.
- Phoon, K.-K., Ed. (2008). *Reliability-based design in geotechnical engineering*, Taylor & Francis, Abingdon, UK.
- Phoon, K.-K. and Kulhawy, F. H. (2002). "Drilled shaft design for transmission structures using LRFD and MRFD." *Deep Foundations 2002 (GSP 116)*, O'Neill, M. W. and Townsend, F. C., Eds., ASCE, Reston, VA, 1006-1017.

- Phoon, K.-K., Kulhawy, F. H. and Grigoriu, M. D. (1995). "Reliability-based design for transmission line structures". *Report TR-105000*, Electric Power Research Institute, Palo Alto, CA.
- Phoon, K.-K., Kulhawy, F. H. and Grigoriu, M. D. (2000). "Reliability-based design for transmission line structure foundations." *Computers and Geotechnics*, 26(3/4), 169-185.
- Phoon, K.-K., Kulhawy, F. H. and Grigoriu, M. D. (2003). "Development of a reliability-based design framework for transmission line structure foundations." *Journal of Geotechnical and Geoenvironmental Engineering*, 129(9), 798-806.
- Rackwitz, R., Lentz, A. and Faber, M. (2005). "Socio-economically sustainable civil engineering infrastructures by optimization." *Structural Safety*, 27(3), 187-229.
- Rackwitz, R. and Fiessler, B. (1978). "Structural reliability under combined load sequences." *Computers and Structures*, 9(5), 489-494.
- Rackwitz, R. (2004). "Optimal and acceptable technical facilities involving risks." *Risk Analysis*, 24(3), 675-695.
- Rackwitz, R. (2002). "Optimization and risk acceptability based on the life quality index." *Structural Safety*, 24(2-4), 297-331.
- Raizer, V. (2004). "Theory of reliability in structural design." *Applied Mechanics Reviews*, 57(1), 1-21.

- Rosen, S. (1994). "Safety and the saving of life: The theory of equilizing differences." *Cost-benefit analysis*. Layard, R. and Glaister, S., Eds., Cambridge University Press, Cambridge, UK, 272-289.
- Rosenblueth, E. (1976). "Towards optimum design through building codes." *Journal of the Structural Division, ASCE*, 102(3), 591-607.
- Rosenblueth, E. (1986). "Optimum reliabilities and optimum design." *Structural Safety*, 3(2) 69-83.
- Rosenblueth, E. and Mendoza, E. (1971). "Reliability optimization in isostatic structures." *Journal of the Engineering Mechanics Division, ASCE*, 97(6), 1625-1642.
- Sanchez-Silva, M. and Rackwitz, R. (2004). "Socioeconomic implications of life quality index in design of optimum structures to withstand earthquakes." *Journal of Structural Engineering*, 130(6), 969-977.
- Sanchez-Silva, M. and Rosowsky, D. V. (2008). "Risk, reliability and sustainability in the developing world." *Proceedings, Institution of Civil Engineers: Structures and Buildings*, 161(4), 189-197.
- Sarma, K. C. and Adeli, H. (1998). "Cost optimization of concrete structures." *Journal of Structural Engineering*, 124(5), 570-578.
- Scott, B., Kim, B. J. and Salgado, M. (2003). "Assessment of current load factors for use in geotechnical load and resistance factor design." *Journal of Geotechnical and Geoenvironmental Engineering*, 129(4), 287-295.

- Shan, S. and Wang, G. G. (2008). "Reliable design space and complete single loop reliability-based design optimization." *Reliability Engineering and System Safety*, 93(8), 1218-1230.
- Song, J. and Der Kiureghian, A. (2003). "Bounds on system reliability by linear programming." *Journal of Engineering Mechanics*, 129(6), 627-636.
- Southern Building Code Congress International (SBCCI) (1999). *Standard building code*. SBCCI, Birmingham, AL.
- Stewart, M. G. (1992). "Modeling human error rates for human reliability analysis of a structural design task." *Reliability Engineering and System Safety*, 36(2), 171-180.
- Terzaghi, K. and Peck, R. B. (1948). *Soil mechanics in engineering practice*, John Wiley & Sons, New York, NY.
- Thoft-Christensen, P. and Baker, M. J. (1982). *Structural reliability theory and its application*, Springer, Berlin, Germany.
- Treloar, G. J., Love, P. E. D., Faniran, O. O. and Iyer-Raniga, U. (2000). "A hybrid life cycle assessment method for construction." *Construction Management and Economics*, 18(1) 5-9.
- Tu, J., Choi, K. K. and Park, Y. H. (1999). "A new study on reliability-based design optimization." *Journal of Mechanical Design*, 121(4), 557-564.
- Turkstra, C. J. (1967). "Choice of failure probabilities." *Journal of the Structural Division, ASCE*, 93(6), 189-200.

- Vesic, A. S. (1975). "Bearing capacity of shallow foundations." *Foundation engineering handbook*, Winterkorn, H. F. and Fang, H. Y., Eds., Van Nostrand Reinhold, New York, NY.
- Vick, S. G. (2002). *Degrees of belief: subjective probability and engineering judgement*. ASCE, Reston, VA.
- Vrouwenvelder, T. (2008). "Treatment of risk and reliability in eurocodes." *Proceedings, Institution of Civil Engineers: Structures & Buildings*, 161(4), 209-214.
- Wang, Y. and Kulhawy, F. H. (2008a). "Economic design optimization of foundations." *Journal of Geotechnical and Geoenvironmental Engineering*, 134(8), 1097-1105.
- Wang, Y. and Kulhawy, F. H. (2008b). "Reliability index for serviceability limit state of building foundations." *Journal of Geotechnical and Geoenvironmental Engineering*, 134(11), 1587-1594.
- Wen, Y. K. and Kang, Y. J. (2001a). "Minimum building life-cycle cost design criteria. I: Methodology." *Journal of Structural Engineering*, 127(3), 330-337.
- Wen, Y. K. and Kang, Y. J. (2001b). "Minimum building life-cycle cost design criteria. II: Applications." *Journal of Structural Engineering*, 338-346.
- Whitman, R. (1984). "Evaluating calculated risk in geotechnical engineering." *Journal of Geotechnical Engineering*, 110(2), 143-188.
- Wong, F. (1985). "Slope reliability and response surface method." *Journal of Geotechnical Engineering*, 111(1), 32-53.

- Xu, B. and Low, B. K. (2006). "Probabilistic stability analyses of embankments based on finite-element method." *Journal of Geotechnical and Geoenvironmental Engineering*, 132(11), 1444-1454.
- Youn, B. D. and Choi, K. K. (2004). "Selecting probabilistic approaches for reliability-based design optimization." *American Institute of Aeronautics and Astronautics Journal*, 42(1), 124-131.
- Zhang, J., Zhang, L. M. and Tang, W. H. (2011). "Reliability-based optimization of geotechnical systems." *Journal of Geotechnical and Geoenvironmental Engineering*, 137(12), 1211–1221.
- Zhang, L. M. and Phoon, K.-K. (2006). "Serviceability considerations in reliability-based foundation design." *Foundation Analysis and Design: Innovative Methods (GSP 153)*, Parsons, R. L., Zhang, L., Guo, W. D., Phoon, K. -K. and Yang, M. Eds., ASCE, Reston, VA, 127-136.